Applied Mathematical Sciences, Vol. 8, 2014, no. 43, 2141 - 2147 HIKARI Ltd, www.m-hikari.com http://dx.doi.org/10.12988/ams.2014.42137 Computing Generators of Second Homotopy Module Using Transformation *Method* Yanita Department of Mathematics, Faculty of Mathematics and Natural Science Universitas Andalas, Kampus Unand Limau Manis, Padang, 25163 Indonesia  Abstract This *article* discusses *about computing generator* of second homotopy module, that obtained from group presentation. Two group presentations can be transformed to each other using transformation. We consider the second homotopy module for both group presentations to compute their generators. Mathematics Subject Classification : 14F35, 14H30, 20F05, 20M05 Keywords: Second homotopy module, transformation, Generator 1 Introduction Let be a presentation for a group G. Then we have the first fundamental group *n* () over . The elements of *n* () are equivalent classes of words *r*. Moreover, we can have a picture over *.A* picture over is an object consist of disjoint arcs labelled by element of *x*, discs labelled by element of *r*, and a boundary disc with a basepoint. A picture over is a spherical picture if all arcs in do not touch the boundary disc. Then we have the second homotopy module *n ()*. The elements of *n ()* are equivalent classes of spherical picture *[]*. *2142 Yanita* Let a group defined group presentation, say . There are some alternations one can make to presentation 2 which result in presentation of a group isomorphic to the original 1 (see *[7]* and *[8]). There* are several transformations for to this case, namely the Tietze transformation and transformation. The definition of transformation will be presented below: Definition 1. (Definition of transformation, see [3], [4], [5]) Let be a presentation define a group . : replacing by r r for some and r word in . 2 : replacing by for some . 3 : replacing by or ( ). 4 : replacing throughout by : a) or b) or c) (this operation consist of exactly one of case a). b). c).) ! : adding or deleting a new generator and new relator . Remark. Operations of , 2, and 3 in the article [5] is called elementary operation. In this article also mentioned that earlier scholars use the term transformation for elementary operations. If the elementary operations are added to 4, then is called transformation. Meanwhile, if the operations which contains only 4 knows as the Nielsen transformation. Operation of ! is the same as "# or "$ in Tietze transformation. The problem of n () is to compute its generator (see [2]). Computing generator of second homotopy module using Tietze transformation methods has been performed (see [10]). 2. The Main Result Theorem 1. Let 1 and 2 %& be a presentation define a group . a. If picture n containing a -disc, so picture n that consist of the same disc with -disc, becomes r r -disc, for some and r word in ' Then generator of n is equal to n . b. If picture n containing of a -disc, so picture n that containing the same disc with -disc, becomes -disc and be connected with -disc ( (. If is generator of n then generator of n , which is derived from , that is, r-disc is replaced with -disc.

Computing generators of second homotopy module 2143 Theorem 2. Let 1 and 2 %& be a presentation define a group . If ) for each )& and * is set of generator of n +( then * is set of generator of n +( too. Proof of this theorem by using operations on picture will be given on section *4.* 3. Picture and Operation on Picture This section will explain the picture and operations on picture, which is referred to [2], [6], and [9]. We introduced the definition of disc hoop, ie, disc which has the arc that begins and ends on the disc. If there is a disc hoop on a picture, then the arc that is hoop on the picture can be eliminated because there is no other meaning topology. Figure 1. Disc hoop A picture in is an object consist of disjoint arcs labeled by element of xdiscs labeled by element of r and a boundary disc with a basepoint (see *[2]* and *[6])*. A picture in is a spherical picture if all arcs in do not touch the boundary disc. Certain basic operation can be applied to a picture (spherical picture) as follows: deletion and insertion floating circle, deletion and insertion floating semicircle, deletion and insertion folding pair and bridge move (see [9]).

2144 Yanita Figure 2. Example of Picture Figure 3. Example of Spherical Picture Figure 4. Bridge Two spherical pictures 1 and 2 are said to be equivalent if either: (a) both are spherical and one can be transformed to the other by a finite number of operation deletion and insertion floating circle, deletion and insertion folding pair and bridge move; or (b) both are not spherical and one can be transformed to the other by a finite number of operation deletion and insertion floating circle, deletion and insertion semicircle, deletion and insertion folding pair and bridge move.

Computing generators of second homotopy module 2145 The equivalent class containing the spherical picture is denoted by []. The equivalent class containing the empty picture (null) is denoted by *[2].* The mirror image for the spherical picture is denoted by *,*. The addition 1 + 2 is defined by drawing 1 and 2. Set of equivalent classes of spherical picture with binary operation [1] + [2] = [1 + 2] form a abelian group under this operation and this abelian group is right *--module,* where the action is given by []r.r (r. denotes the element of represented by *r).*

This module is called the second homotopy module of , denoted by *n* (). A set P of spherical pictures over will be called a generating set of pictures if * generates the *--module n* () (see [1]) . It follow *[2],* that P is generating set if and only if every spherical picture over can be transformed to empty picture by operations: bridge moves, insertion/deletion of floating circles, insertion/deletion of folding pairs, insertion/deletion of pictures from */*.* Consider a collection *0* of spherical pictures. Now, we define two extended operation on pictures as follows : 1). (Deletion of an *0-picture)* If there is a simple closed path in a picture such that the part of the picture enclosed by the simple closed path is a copy of a spherical picture. 2). (Insertion of an *0-picture)* The opposite of 1). Two pictures will be said to be equivalent (relative *0)* if either: a). the pictures are both spherical and one can be transformed to the other by a finite number of operation deletion and insertion floating circle, deletion and insertion folding pair, bridge move, and deletion and insertion 0-picture; or b). the *picture* are *not* both spherical and one can be transformed to the other by a finite number of *operations* deletion and insertion floating circle, deletion and insertion floating semicircle, deletion and insertion folding pair, bridge move and deletion and insertion 0-picture (see [9]). 4. Proof of Theorem Proof of Theorem 1. a. We consider that the picture containing ,disc will turn into picture that containing r r ,disc, as shown below:

2146 Yanita Then we did the bridge move operation on the picture . We have that picture n containing disc hoop, ie, r r ,disc, then we can eliminate arc that is labeled word r. Therefore, we have picture equal to . b. Analogous to part a., but picture containing ,disc will turn into picture that containing ,disc, as shown below: We can apply the bridge move operation to simplify the picture . If there are discs hoop, so the arcs that is hoop on the picture can be eliminated. We apply these operations until no longer arcs in the opposite direction of the same labelled. Proof of Theorem 2. Let * set generator of n , so spherical picture of * is labelled with ' If &, then * set of generator of n , if not then there are picture n derived from . Supposed that disc is labelled -disc is replaced by -disc. Consequently, we have the same picture but different the arrow direction of arcs, so * set of generator of n .

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Received: February 19, 2014

1 Introduction Let b t be a presentation for a group *G* . Then we have the first fundamental group n ( ) over b t . The elements of n ( ) are equivalent classes of words *fr* . Moreover, we can have a picture over . *A* picture over is an object consist of disjoint arcs labelled by element of *x* , discs labelled by element of *r* , and a boundary disc with a basepoint. A picture over is a spherical picture if all arcs in do not touch the boundary disc. Then we have the second homotopy module *n ().* The elements of *n ()* are equivalent classes of spherical picture *[ ].* Let a group defined group presentation, say . There are some alternations one can make to presentation 2 which result in presentation of a group isomorphic to the original 1 (see *[7]* and *[8]). There* are several transformations for to this case, namely the Tietze transformation and transformation. The definition of transformation will be presented below: Definition 1. (Definition of transformation, see [3], [4], [5]) Let b t be a presentation define a group . : replacing by r r for some b and r word in . 2 : replacing by for some b . 3 : replacing by or ( b ). 4 : replacing throughout by : a) or b) or c) ( this operation consist of exactly one of case a). b). c). ) ! : adding or deleting a new generator and new relator . Remark . Operations of , 2, and 3 in the article [5] is called elementary operation. In this article also mentioned that earlier scholars use the term transformation for

elementary operations. If the elementary operations are added to 4, then is called transformation. Meanwhile, if the operations which contains only 4 knows as the Nielsen transformation. Operation of ! is the same as "# or "$ in Tietze transformation. The problem of n ( ) is to compute its generator (see [2]). Computing generator of second homotopy module using Tietze transformation methods has been performed (see [10]). 2. The Main Result Theorem 1. Let 1 b t and 2 %& t be a presentation define a group . a. If picture n containing a -disc, so picture n that consist of the same disc with -disc, becomes r r -disc, for some b and r word in b' Then generator of n is equal to n . b. If picture n containing of a -disc, so picture n that containing the same disc with -disc, becomes -disc and be connected with -disc ( b ( . If is generator of n then generator of n , which is derived from , that is, r-disc is replaced with -disc.

Theorem 2. Let 1 b t and 2 %& t be a presentation define a group . If ) for each )& b and * is set of generator of n + ( then * is set of generator of n + ( too. Proof of this theorem by using operations on picture will be given on section *4.* 3. Picture and Operation on Picture This section will explain the picture and operations on picture, which is referred to [2], [6], and [9]. We introduced the definition of disc hoop , ie, disc which has the arc that begins and ends on the disc. If there is a disc hoop on a picture, then the arc that is hoop on the picture can be eliminated because there is no other meaning topology. Figure 1. Disc hoop A picture *in b t* is an object consist of disjoint arcs labeled by element of x discs labeled by element of r and a boundary disc with a basepoint (see *[2]* and *[6]).* A picture *in b t* is a spherical picture if all arcs in do not touch the boundary disc. Certain basic operation can be applied to a picture (spherical picture) as follows: deletion and insertion floating circle , deletion and insertion floating semicircle , deletion and insertion folding pair and bridge move (see [9]).

Figure 2. Example of Picture Figure 3. Example of Spherical Picture Figure 4. Bridge Two spherical pictures 1 and 2 are said to be equivalent if either: (a) both are spherical and one can be transformed to the other by a finite number of operation deletion and insertion floating circle, deletion and insertion folding pair and bridge move; or (b) both are not spherical and one can be transformed to the other by a finite number of operation deletion and insertion floating circle, deletion and insertion semicircle, deletion and insertion folding pair and bridge move. The equivalent class containing the spherical picture is denoted by [ ]. The equivalent class containing the empty picture (null) is denoted by *[2].* The mirror image for the spherical picture is denoted by , . The addition 1 + 2 is defined by drawing 1 and 2 . Set of equivalent classes of spherical picture with binary operation [ 1 ] + [ 2 ] = [ 1 + 2 ] form a abelian group under this operation and this abelian group is right - -module, where the action is given by [ ] *r . fr* ( r . denotes the element of represented by *r* ). This module is called the second homotopy module of , denoted by *n* ( ). A set P of spherical pictures over will be called a generating set of pictures if *f* * generates the - -module n ( ) (see [1]) . It follow *[2],* that P is generating set if and only if every spherical picture over can be transformed to empty picture by operations: bridge moves, insertion/deletion of floating circles, insertion/deletion of folding pairs, insertion/deletion of pictures from */** . Consider a collection *0* of spherical pictures. Now, we define two extended operation on pictures as follows : 1). (Deletion of an *0 -picture)* If there is a simple closed path in a picture such that the part of the picture enclosed by the simple closed path is a copy of a spherical picture. 2). (Insertion of an *0* -picture) The opposite of 1). Two pictures will be said to be equivalent (relative *0* ) if either: a). the *pictures* are both spherical and one can be transformed to the other by a finite number of *operation* deletion and insertion floating circle, deletion and insertion folding pair, bridge move, and deletion and insertion *0 -picture;* or b). the picture are not both spherical and one can be transformed to the other by a finite number of operations deletion and insertion floating circle, deletion and insertion floating semicircle, deletion and insertion folding pair, bridge move and deletion and insertion *0 -picture* (see *[9]). 4.* Proof of Theorem *Proof of* Theorem 1. a. We consider that the picture containing , disc will turn into picture that containing r r , disc, as shown below:

Then we did the bridge move operation on the picture . We have that picture n containing disc hoop, ie, r r , disc, then we can eliminate arc that is labeled word r . Therefore, we have picture equal to . b. Analogous to part a., but picture containing , disc will turn into picture that containing , disc, as shown below: We *can* apply *the* bridge move operation to *simplify* the picture . If there are discs hoop, so the arcs that is hoop on the picture can be eliminated. We apply these operations until no longer arcs in the opposite direction of the same labelled.

Proof of Theorem 2. Let * set generator of n , so spherical picture of * is labelled with b' If b& , then * set of generator of n , if not then there are picture n derived from . Supposed that disc is labelled - disc is replaced by - disc. Consequently, we have the same picture but different the arrow direction of arcs, so * set of generator of n .