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To cite this article: D Devianto *et al* 2019 *J. Phys.: Conf. Ser.* **1397** 012065

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The Infinitely Divisible Characteristic Function of Compound Poisson Distribution as the Sum of Variational Cauchy Distribution

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Abstract. The new particular compound Poisson distribution is introduced as the sum of independent and identically random variables of variational Cauchy distribution with the number of random variables has Poisson distribution. This compound Poisson distribution is characterized by using characteristic function that is obtained by using Fourier-Stieltjes transform. The infinite divisibility of this characteristic function is constructed by introducing the specific function that satisfied the criteria of characteristic function. This characteristic function is employing the properties of continuity and quadratic form in term of real and non-negative function such that its convolution has the characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution.

1. Introduction

A compound Poisson distribution is a sum of independent and identically random variables for any distributions where the number of these random variables has Poisson distribution. The random variable of compound Poisson distribution is defined in the mathematical term as $S=X_1+X_2+\dots+X_N$ where the random variable N has Poisson distribution and independent and identically random variables $X_{i=1,2,\dots,N}$ have certain distribution. In the case random variable X_i has variational Cauchy distribution then it is said the random variable S has compound Poisson distribution as the sum of variational Cauchy distribution.

The Poisson distribution and Cauchy distribution are well defined probability distribution where Sato [1] has explained their infinite divisibility. The infinite divisibility can be identified from its distribution, characteristic function and random variable. Steutel and Harn [2] have stated the infinite divisibility of random variable S with distribution function F can be divided into m random variables such that $S=X_1+X_2+\dots+X_m$ where the random variable $X_{i=1,2,\dots,m}$ as independent and identically random variables with distribution function F_m , this is well known as m -fold convolution of distribution function of F_m that is $F=F_m * F_m * \dots * F_m$ for m times. Besides the terminology of a convolution for infinitely divisible distribution, Lukacs [3] has expressed the infinite divisibility by using characteristic function as Fourier-Stieltjes transform, where the characteristic function is defined as $\phi(t)=E(\exp(itX))$ for a random variable X where i as imaginary unit and $t \in (-\infty, \infty)$. The definition of infinitely divisible characteristic function can be also referred from Artakis [4] or Mainardi and Rogosin [5], the characteristic function $\phi(t)$ is said to be infinitely divisible if for every positive integer number m , there exists a characteristic function



$\phi_m(t)$ such that $\phi(t) = (\phi_m(t))^m$. The existence of characteristic function $\phi_m(t)$ could be examined by using necessary and sufficient condition for a function to be characteristic function by Bochner's theorem in Lukacs [3], that is the criteria of $\phi_m(0)=1$ and the function $\phi_m(t)$ non-negative definite, where the condition of non-negative definite is satisfied by continuity and quadratic form of the function $\phi_m(t)$.

The concept of infinitely divisible distribution can be applied to develop the theory of compound distribution by using its property of characteristic function, that is also known as the sum of random variables as compounding distribution constructed as the conditional convolution of m independent and identically random variables. Some theories of a convolution for certain distribution have introduced by Devianto et. al [6,7] as convolution of the generated random variable from exponential and its variation with stabilizer constant. Furthermore, the infinite divisibility of half-Cauchy distribution and its properties explained in Diedhiou [8] and the infinite divisibility of normed product of Cauchy distribution by Takano [9]. Furthermore, Serfozo [10] has discussed the theory of compound Poisson approximation for sums of random variables, while Cekanavicius and Vellaisamy [11] and Barbour et. al [12] have another setting on convergence approximation of compound Poisson distribution via Stein's method.

The property of discrete compound Poisson process on its infinite divisibility and convergence has introduced by Zhang and Li [13]. Although the convergence in distribution of compound Poisson process has introduced as the theoretical approach, but the form of density of compound Poisson distribution also has interested to deliver, where Duval [14] has estimated the density of compound Poisson processes from discrete data, while Lindo et. al [15] has used nonparametric estimation for compound Poisson process via variational analysis on measures.

The property of compound Poisson distribution has introduced by the previous researcher influencing many rules in application particularly on insurance risk models distribution such as Zhang et.al [16] and Hu et. al [17] have introduced discrete compound Poisson model with applications to risk theory. Furthermore, Willmout [18] have reviewed Lundberg approximations for compound distributions with insurance applications. While the application of convolution theory to characteristic function also has explained by Devianto [19] in term of uniform continuity properties of characteristic function from the convolution of exponential distribution with stabilizer constant, and its class of infinitely divisible exponential distribution in Devianto [20]. The theoretical concept and application of compound distribution have the strong position to confirm that characteristic function has important rules on deriving the property of compound distribution especially on its infinite divisibility. However, the summary of recent results does not cover the characterization of specific compound Poisson distribution as the sum of variational Cauchy distribution on its property of characteristic function, then this paper will give exploration on development of characteristic function and its property of infinitely divisible, quadratic form, continuity and no real zeros attributed as the main findings.

2. The Characteristic Function of Compound Poisson Distribution as the Sum of Variational Cauchy Distribution

The property of compound Poisson distribution as the sum of variational Cauchy distribution is constructed by using characterization of its characteristic function. First, it is introduced the Poisson distribution and variational Cauchy distribution in the sense of probability distribution and its characterizations on moment generating function and characteristic function. The Poisson distribution is a discrete probability distribution for counts N of events that occur randomly in a given interval of time with λ is the mean number of events per interval. The probability distribution is given by equation

$$P(N = n) = \frac{e^{-\lambda} \lambda^n}{n!} \quad (1)$$

where $n = 1, 2, 3, \dots$ and $\lambda > 0$. This distribution is noted as Poisson distribution with a parameter (λ).

The moment generating function of Poisson distribution is obtained as

$$M_N(t) = E(e^{tN}) = \exp(\lambda(e^t - 1)) \quad (2)$$

for $t \in (-\infty, \infty)$. The most property of Poisson distribution is in the form of characteristic function as follows

$$\phi_N(t) = E(e^{itN}) = \exp(\lambda(e^{it} - 1)) \quad (3)$$

for $t \in (-\infty, \infty)$.

The variational Cauchy distribution is constructed by setting parameter γ multiplied with its random variable in standard Cauchy distribution, so that it can be defined in the following term of probability density function

$$f(x) = \frac{\sqrt{\gamma}}{\pi(1 + \gamma x^2)} \quad (4)$$

for $\gamma > 0$ and $-\infty < x < \infty$. This distribution is noted as variational Cauchy distribution with a parameter (γ). The special characterizations of variational Cauchy distribution are undefined for expected value and variance, meanwhile there is also no existence of moment generating. Since the characteristic function is, always exist for every distribution, then the characteristic function of variational Cauchy distribution is obtained as follows

$$\phi_X(t) = E(e^{itX}) = \exp\left(\frac{-|t|}{\sqrt{\gamma}}\right) \quad (5)$$

for $t \in (-\infty, \infty)$.

The characteristic function of Poisson distribution and Cauchy distribution have expressed into canonical representation of Levy-Khintice in Lukacs [3], then both distributions are infinitely divisible. Furthermore, the infinite divisibility of compound Poisson distribution as the sum of variational Cauchy distribution has to express some important properties on its characteristic function as the main concern on these current results. The characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution is given in the following proposition. The following theoretical concept of compounding distribution is stated by using the random variable $S = X_1 + X_2 + \dots + X_N$ as a compound distribution where the random variable N has Poisson distribution and random variables $X_{i=1,2,\dots,N}$ have variational Cauchy distribution with same parameters. This distribution is noted as compound Poisson distribution as the sum of variational Cauchy distribution with parameters (λ, γ) .

Proposition 2.1. Let $S = X_1 + X_2 + \dots + X_N$ be a random variable of compound Poisson distribution as the sum of variational Cauchy distribution, then characteristic function of random variable S is in the following form

$$\phi_S(t) = \exp\left(\lambda \left(\exp\left(\frac{-|t|}{\sqrt{\gamma}}\right) - 1\right)\right) \quad (6)$$

for $t \in (-\infty, \infty)$, $\lambda > 0$ and $\gamma > 0$.

Proof. It is used definition of characteristic function and the linearity property of expectation, and then it is obtained the characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution as follows

$$\phi_S(t) = E(\exp(itS)) = E\left(E\left(\exp\left(it \sum_{i=1}^N X_i\right) \middle| N\right)\right) = M_N(\ln(\phi_X(t))). \quad (7)$$

It is substituted the characteristic function of variational Cauchy distribution in Equation (5) and the moment generating function of Poisson distribution on Equation (2) to Equation (7), then it is obtained

$$\phi_S(t) = M_N\left(\ln\left(\exp\left(\frac{-|t|}{\sqrt{\gamma}}\right)\right)\right) = \exp\left(\lambda \left(\exp\left(\frac{-|t|}{\sqrt{\gamma}}\right) - 1\right)\right). \quad (8)$$

The Proposition 2.1 has shown that compound Poisson distribution as the sum of variational Cauchy distribution has a unique characteristic function, that is very varying from the characteristic function of the sum of variational Cauchy distribution such as in Devianto [21] or the property of characteristic function as normed product of Cauchy density in Takano [9], and this new type of characteristic function has different property of characteristic function of half-Cauchy distribution in Diedhiou [8] and it is not concluded in generalized Cauchy family of distributions in Alzaatreh et. al [22]. In addition, it is unlikely a compound Poisson process such as in Franciszek et. al [23] or Seri and Choirat [24], but this characteristic function is governed by using Poisson compounding with certain distribution. This is to confirm the uniqueness of characteristic function for difference formula of compound Poisson distribution.

3. The Characterization of Infinitely Divisible Characteristic Function of Compound Poisson Distribution as the Sum of Variational Cauchy Distribution

The main results of this paper are stated in the following proposition and theorem by explaining the characterization of characteristic function and its property of infinite divisibility. The properties of the sum of random variables as compound distribution can also be identified by using its characteristic function in the term of infinite divisibility. Furthermore, the infinite divisibility of compound Poisson distribution as the sum of variational Cauchy distribution is constructed by using property of characteristic function. First, let us define a function $\phi_{S_m}(t)$ as a corresponding function to the random variable S_m with the cumulative distribution function F_{S_m} . Now, it is defined the function $\phi_{S_m}(t)$ as follows

$$\phi_{S_m}(t) = \left(\exp \left(\lambda \left(\exp \left(\frac{-|t|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} \quad (9)$$

for any integer number m where λ as parameter of Poisson distribution and γ as parameter of variational Cauchy distribution. The next propositions are properties of function $\phi_{S_m}(t)$ related to necessary and sufficient condition in Bochner's theorem for a function $\phi_{S_m}(t)$ to be a characteristic function.

Proposition 3.1. The function $\phi_{S_m}(t)$ is continuous.

Proof. The continuity of function $\phi_{S_m}(t)$ is explained by using the condition of uniform continuity, that is for every $\varepsilon > 0$ there exists $\delta > 0$ such that $|\phi_{S_m}(t_1) - \phi_{S_m}(t_2)| < \varepsilon$ for $|t_1 - t_2| < \delta$ where δ depends only on ε . Hence, it is used definition of function $\phi_{S_m}(t)$ as in Equation (9) to have the following equation

$$|\phi_{S_m}(t_1) - \phi_{S_m}(t_2)| = \left| \left(\exp \left(\lambda \left(\exp \left(\frac{-|t_1|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} - \left(\exp \left(\lambda \left(\exp \left(\frac{-|t_2|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} \right|. \quad (10)$$

Then let us define $h = t_1 - t_2$ and then it is applied to the Equation (10), so that for $h \rightarrow 0$, it is obtained the following limit

$$\lim_{h \rightarrow 0} \left| \left(\exp \left(\lambda \left(\exp \left(\frac{-|h+t_2|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} - \left(\exp \left(\lambda \left(\exp \left(\frac{-|t_2|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} \right| = 0. \quad (11)$$

This holds for $\delta < \varepsilon$ where $|\phi_{S_m}(t_1) - \phi_{S_m}(t_2)| < \varepsilon$ for $|t_1 - t_2| < \delta$ and δ depends only on ε . Then $\phi_{S_m}(t)$ satisfied the condition of uniformly continuous, then it is concluded that the function $\phi_{S_m}(t)$ is continuous.

Propositions 3.2. The function $\phi_{S_m}(t)$ satisfies the condition of quadratic form.

Proof. It is used the Equation (9) for variable $t_j - t_l$ where j and l as positive integer numbers such that we have the following doubled summation

$$\begin{aligned} \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \phi_{S_m}(t_j - t_l) &= \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \left(\exp \left(\lambda \left(\exp \left(\frac{-|t_j - t_l|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right)^{\frac{1}{m}} \\ &= \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \left(\exp \left(\frac{\lambda}{m} \left(\exp \left(\frac{-|t_j - t_l|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right) \end{aligned} \quad (12)$$

for any complex number c_j and c_l with parameters $(\lambda/m, \gamma)$. Next, based on the nature of absolute value of $|t_j - t_l|$ then it can be written the set of the following condition, the first $|t_j - t_l| = t_j - t_l$ for $t_j - t_l > 0$ and the second $|t_j - t_l| = -t_j + t_l$ for $t_j - t_l < 0$. Next, the Equations (12) is divided into two parts as the following forms

(i) The function $\phi^+(t_j, t_l) = \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \exp \left(\frac{\lambda}{m} \left(\exp \left(\frac{-t_j + t_l}{\sqrt{\gamma}} \right) - 1 \right) \right)$ for $t_j - t_l > 0$.

(ii) The function $\phi^-(t_j, t_l) = \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \exp \left(\frac{\lambda}{m} \left(\exp \left(\frac{t_j - t_l}{\sqrt{\gamma}} \right) - 1 \right) \right)$ for $t_j - t_l < 0$.

Next, it will be explained the real and non-negative property of Equation (12) for both Part (i) and Part (ii). First, it is defined a random variable Y as compound Poisson distribution as the sum of variational Cauchy distribution with parameters $(\lambda/m, \gamma)$, and it is used the property of complex conjugate, then it is obtained the Part (i) from Equation (12) satisfied the following equation

$$\begin{aligned} \phi^+(t_j, t_l) &= \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \exp \left(\frac{\lambda}{m} \left(\exp \left(\frac{-t_j + t_l}{\sqrt{\gamma}} \right) - 1 \right) \right) \\ &= \left(\sum_{1 \leq j \leq n} c_j E(\exp(it_j(-Y))) \right) \overline{\left(\sum_{1 \leq l \leq n} c_l E(\exp(it_l(-Y))) \right)} \\ &= \left| \sum_{1 \leq j \leq n} c_j E(\exp(it_j(-Y))) \right|^2 \end{aligned} \quad (13)$$

The property of the real and non-negative in Part (ii) from Equation (12) can be seen in the following equation

$$\begin{aligned} \phi^-(t_j, t_l) &= \sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \exp \left(\frac{\lambda}{m} \left(\exp \left(\frac{t_j - t_l}{\sqrt{\gamma}} \right) - 1 \right) \right) \\ &= \left(\sum_{1 \leq j \leq n} c_j E(\exp(it_j Y)) \right) \overline{\left(\sum_{1 \leq l \leq n} c_l E(\exp(it_l Y)) \right)} \\ &= \left| \sum_{1 \leq j \leq n} c_j E(\exp(it_j Y)) \right|^2 \end{aligned} \quad (14)$$

From Equation (13) and Equation (14), it is concluded that

$$\sum_{1 \leq j \leq n} \sum_{1 \leq l \leq n} c_j \bar{c}_l \phi_{S_m}(t_j - t_l) \geq 0, \quad (15)$$

Then it is proved that the function $\phi_{S_m}(t)$ as real and positively defined function where the quadratic form has non-negative values.

Proposition 3.3. The function $\phi_{S_m}(t)$ is characteristic function.

Proof. Based on Proposition 3.1 and Proposition 3.2, the function $\phi_{S_m}(t)$ satisfied the condition continuity and non-negative definite function, in addition together with condition of $\phi_{S_m}(0)=1$ then the function $\phi_{S_m}(t)$ satisfied the criteria of characteristic function on Bochner's theorem. It is clear that the function $\phi_{S_m}(t)$ is a characteristic function.

The Equation (9) has property of continuity and satisfied the quadratic form such as in Proposition 3.1 and Proposition 3.2 respectively, and then Proposition 3.3 established the function in Equation (9) is characteristic function. The existence of this new particular of compound Poisson distribution as the sum of variational Cauchy distribution has established the property on its characteristic function. This new characteristic function has a specific form that differs from compound Poisson process in Crescenzo et. al [25] or Zhang and Li [13]. The certain distribution of variational Cauchy distribution in Poisson compounding has governed the uniqueness of characteristic function with the property of continuity and quadratic form. These results will be used to establish the property of infinitely divisible of the certain compound Poisson distribution as in Theorem 3.4, and the property of no real zeroes of characteristic function as in the Theorem 3.5.

Theorem 3.4. The compound Poisson distribution as the sum of variational Cauchy distribution is infinitely divisible distribution.

Proof. The property of infinitely divisible characteristic function is shown by using the property of characteristic function $\phi_{S_m}(t)$ such that satisfies $\phi_S(t) = (\phi_{S_m}(t))^m$ for $\phi_S(t)$ as the characteristic function of compound Poisson distribution. Based on Proposition 3.3, there exists a characteristic function $\phi_{S_m}(t)$ such that it is obtained

$$(\phi_{S_m}(t))^m = \left(\exp \left(\lambda \left(\exp \left(\frac{-|t|}{\sqrt{\gamma}} \right) - 1 \right) \right)^{\frac{1}{m}} \right)^m = \exp \left(\lambda \left(\exp \left(\frac{-|t|}{\sqrt{\gamma}} \right) - 1 \right) \right) = \phi_S(t), \quad (16)$$

for any positive integer number m , the function $\phi_S(t)$ is a characteristic function from compound Poisson distribution as the sum of variational Cauchy distribution with parameters (λ, γ) , so that it is an infinitely divisible characteristic function.

The random variable S_m has corresponding characteristic function $\phi_{S_m}(t)$ of compound Poisson distribution as the sum of variational Cauchy distribution with the parameter $(\lambda/m, \gamma)$, where λ/m as the mean number of events for every interval and γ as the parameter in variational Cauchy distribution. This result confirmed the compound Poisson distribution as the sum of variational Cauchy distribution is also infinitely divisible, where the property of infinite divisibility also satisfies for Poisson distribution and variational Cauchy distribution.

Theorem 3.5. The characteristic function $\phi_S(t)$ has no real zeroes.

Proof. Based on Theorem 3.4, the characteristic function $\phi_S(t)$ is infinitely divisible, then the function $|\phi_{S_m}(t)|^2$ is a characteristic function satisfied the condition $|\phi_{S_m}(t)|^2 = |\phi_S(t)|^{2/m}$ for any positive integer m . Let us consider the defined function $g(t)$ in the following form

$$g(t) = \lim_{m \rightarrow \infty} |\phi_{S_m}(t)|^2 = \lim_{m \rightarrow \infty} |\phi_S(t)|^{\frac{2}{m}} = \lim_{m \rightarrow \infty} \left| \exp \left(\lambda \left(\exp \left(\frac{-|t|}{\sqrt{\gamma}} \right) - 1 \right) \right) \right|^{\frac{2}{m}}, \quad (17)$$

It is assumed that the function $g(t)$ has only two values, that are 0 or 1. Then it is obtained $g(t)=1$ whenever $\phi_S(t) \neq 0$ and $g(t) = 0$ for all t for which $\phi_S(t) = 0$. Because the characteristic function $\phi_{S_m}(t)$ is continuous based on Proposition 3.1, so that it can easily also be proved that the characteristic function $\phi_S(t)$ is continuous, and as well as it holds for Proposition 3.2, so that $\phi_{S_m}(0) = 1$, then $\phi_S(0) = 1$ therefore $\phi_S(t) \neq 0$.

The property of no real zeroes on characteristic function has spread the characterization of compound Poisson distribution as the sum of variational Cauchy distribution into the specific property. The previous result on Poisson compounding in Zhang and Li [13] and Crescenzo et. al [25] have introduced characterizations of discrete compound Poisson distribution with a Poisson subordinator, but these results does not cover the property of no real zeros of its characteristic function. The result on Theorem 3.5 has completed the new property of this compound Poisson distribution, this characterization is also expressed graphically in Figure 1. The Figure 1 Part (a) describes the characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution with various parameter γ , the curve more platykurtic for γ tends to infinity. The Figure 2 Part (b) describes the curve more leptokurtic for λ tends to infinity, where the both curves have the vertical axis with value 1 for t equals zero. The shape of characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution has exhibited smooth lines as its property of continuity, symmetrical and never has real zeros as infinitely divisible characteristic function.

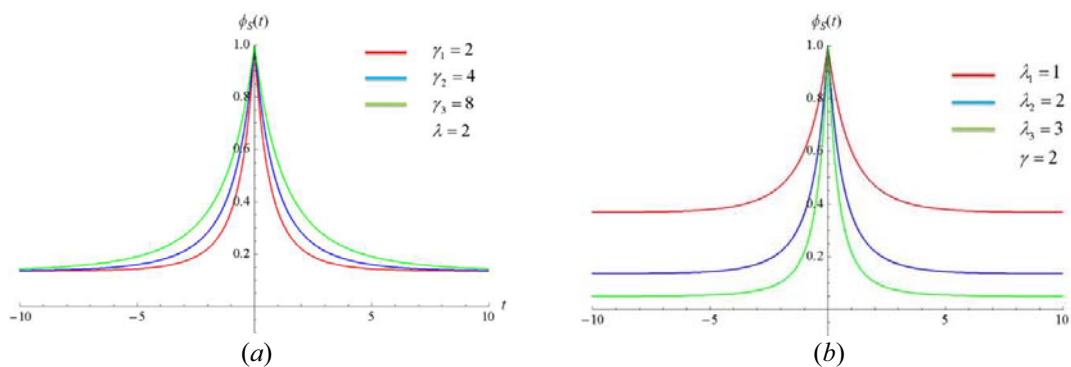


Figure 1. The curves of characteristic $\phi_S(t)$, Part (a) the curve with $\lambda=2$ and various parameters γ , Part (b) the curve with $\gamma=2$ and various parameters λ .

4. Conclusion

In this paper it is constructed the new particular compound Poisson distribution as the sum of variational Cauchy distribution with random variable $S=X_1+X_2+\dots+X_N$ where X_1, X_2, \dots, X_N are independent and identically random variables of variational Cauchy distribution and the random variable N has Poisson distribution. The characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution is obtained by using Fourier-Stieltjes transform as $\phi_S(t)$ with parameter (λ, γ) for λ as parameter of Poisson distribution and γ as parameter of variational Cauchy distribution. It is provided a specific function corresponding to a Poisson compounding process for a distribution such that satisfied the criteria of characteristic function employing the properties of uniform continuity and quadratic form in the term of real and non-negative function. This characteristic function $\phi_S(t)$ satisfies the necessary and sufficient conditions to be infinitely divisible of compound Poisson distribution as the sum of variational Cauchy distribution. In addition, the characteristic function of compound Poisson distribution as the sum of variational Cauchy distribution has no imaginary part and no real zeros, it is also more platykurtic for γ tends to infinity and more leptokurtic for λ tends to infinity.

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