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RESEARCH ARTICLE

Other Properties and Presentation of Group from Kronecker Product on the Representation Quaternion Group

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The group was obtained by apply Kronecker product on the demonstration of the quaternion group is a finite group with 32 order. The elements of this group are 4×4 matrices and the group is a non-abelian. In the first results, it's presented that the group was a solvable group. In this paper, other properties will be presented and will also be given a presentation of the group. This property is associated with a series normal subgroup and the cyclic subgroup. It's shown that the presentation of group have 4 generators and 5 relations.

Keywords: Series Normal Subgroup, Cyclic Subgroup, Group Presentation.

1. INTRODUCTION

We review group G^i in Ref. [1]. The group has 32 elements, where elements are 4×4 matrices. Among the 73 proper subgroups, 19 subgroups with 2 orders, 15 subgroups with 4 orders, 34 subgroups with 8 orders, and 4 subgroups with 16 orders. In addition, there are 25 cyclic subgroups; 19 subgroups with 2 orders and 6 subgroups with 4 orders.

That group has 32 conjugation classes and every element is a conjugation class.[a] Thus, $K_{A_i} = \{A_i\}$ dengan $i = 1, 2, \ldots, 32$. Elements of G that commute with every element $A_1 = I_4$ (identity matrix with 4 orders) and $A_2 = -A_1$, so we have central of G or $Z(G) = \{A_1, A_2\}$.

The paper is organised as follows. In Section 2, we list properties of group G. In Ref. [1], it has been explained that group G has solvable and this section other properties are given. These properties use series normal subgroup and cyclic subgroup. In Section 3 we explain about presentation group of G.

2. PROPERTIES OF GROUP *G*

It has been stated that there are 25 cyclic subgroups of G. All factor groups that occur from cyclic subgroups are not cyclic. This proves that the group G is not metacyclic.[b]

Since all subgroups of G are normal, there are many series normal subgroups in this group. However, there is no cyclic factor group from these series. This proves that this group is not supersolvable[c] and not polycyclic.[d]

It's known that $Z(G) = \{A_1, A_2\}$, then defined $Z_1(G) = Z(G)$ and $Z_{i+1}(G) = [Z_i(G), G]$, where $[Z_i(G), G] = \{[A, B] | A \in Z_i(G), B \in G\}$. Consider that $Z_1(G) = Z(G) = \{A_1, A_2\}$, $Z_2(G) = [Z_1(G), G] = \{A_1, A_2\}$, $Z_3(G) = [Z_2(G), G] = \{A_1, A_2\}, \ldots, Z_n(G) = [Z_{n-1}(G), G] = \{A_1, A_2\}$. It's shown that there are no integers n such that $Z_{n+1}(G) = \{A_1\}$, so this group is not nilpotent.[e]

3. PRESENTATION OF *G*

Group presentations are a way of defining groups using generators and relations. These generators and relations are related to the elements in the group. If P be a presentation of group G, then each element in G can be presented with one of the generator and relation in the presentation [6]. Generators and relations in this presentation are related

[a] Let x is element of group G, then conjugation class of x, denoted by K_x, is set of elements of G that conjugate with x. (Element x and y of G are said conjugate if there is g of G such that $gxg^{-1} = y$) [2].

[b] If there is a normal subgroup cyclic M of group G and factor group G/M is cyclic too, so we called G is metacyclic [3].

[c] Let $H_i, i = 0, 1, \ldots, k$ are subgroup normal in group G, if there is a series normal subgroup $\{e\} = H_0 \subset H_1 \subset H_2 \subset \cdots \subset H_k = G$, and H_{i+1}/H_i is cyclic, then G is supersolvable [2].

[d] Let $H_i, i = 0, 1, \ldots, k$ are subgroup normal in group G, if there is a series normal subgroup $\{e\} = H_0 \subset H_1 \subset H_2 \subset \cdots \subset H_k = G$ where for each $0 \le i < k$, H_i is normal in H_{i+1}, and H_{i+1}/H_i is cyclic for each $i > 0$, then G is polycyclic [5].

[e] A group G is nilpotent if there is a integer n such that $Z_{n+1}(G) = \{e\}$ [6].

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to a term called "word." Therefore, first, the definition of word is given.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of distinct elements and $X^{-1} = \{{x_1}^{-1}, {x_2}^{-1}, \ldots, {x_n}^{-1}\}$ be a set of elements, distinct from each other and from the element of X. Define $X^{\pm 1} = X \cup X^{-1}$. A word W is finite string with form $x_1^{\varepsilon_1} x_2^{\varepsilon_2} \ldots x_{n-1}^{\varepsilon_{n-1}} x_n^{\varepsilon_n}$, $n \geq 0$, $x_i \in X$ and $\varepsilon_i = \pm 1$, $i = 1, 2, \ldots, n$. We can also be called a word is string of alphabet from $X^{\pm 1}$. A word W that are n-tuples are supposed to have length n, symbolized by $|W| = n$. If the word has length 0, or $|W| = 0$, will be symbolized by 1 and the word is called empty word.

Let $W = x_1^{\varepsilon_1} x_2^{\varepsilon_2} \ldots x_{n-1}^{\varepsilon_{n-1}} x_n^{\varepsilon_n}$. Invers of W, denoted by W^{-1}, and defined $W^{-1} = x_n^{-\varepsilon_n} x_{n-1}^{-\varepsilon_{n-1}} \ldots x_2^{-\varepsilon_2} x_1^{-\varepsilon_1}$. If $W_1 = x_{i1}^{\varepsilon_1} \ldots x_{im}^{\varepsilon_m}$ and $W_2 = x_{j1}^{\varepsilon_1} \ldots x_{jn}^{\varepsilon_n}$ are word in $X^{\pm 1}$, then $W_1 W_2 = x_{i1}^{\varepsilon_1} \ldots x_{im}^{\varepsilon_m} x_{j1}^{\varepsilon_1} \ldots x_{jn}^{\varepsilon_n}$. It's clear that $|W| = |W^{-1}|$ and $|W_1 W_2| = |W_1| + |W_2|$.

EXAMPLE 3.1. Let $X = \{a, b, c\}$. We have $X^{-1} = \{1/a, 1/b, 1/c\}$ and $X^{\pm 1} = \{a, b, c, 1/a, 1/b, 1/c\}$. Some word in $X^{\pm 1}$ are

$$W_1 = c^{-1} aabc^{-1} cb$$

$$W_2 = c^{-1} aabc$$

$$W_3 = c^{-1} aabbc^{-1}$$

[i] Group $\mathcal{G} = \left\{ A_k = [a_{ij}] \middle| i, j = 1,2,3,4, k = 1,2, \ldots, 32 \right\}$ where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}, A_4 = \begin{bmatrix} 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, A_7 = \begin{bmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{bmatrix}, A_8 = \begin{bmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{bmatrix},$$

$$A_9 = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}, A_{10} = \begin{bmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{bmatrix}, A_{11} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, A_{14} = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix}, A_{15} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, A_{16} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$A_{17} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, A_{18} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, A_{19} = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{bmatrix}, A_{20} = \begin{bmatrix} 0 & 0 & -i & 0 \\ i & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, A_{23} = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, A_{24} = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}$$

$$A_{25} = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{bmatrix}, A_{26} = \begin{bmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, A_{27} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, A_{28} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_{29} = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, A_{30} = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, A_{31} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, A_{32} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Consider that $|W_1| = 8$, $|W_2| = 5$, and $|W_3| = 6$. Word of form $x_i x_i^{-1}$ or $x_i^{-1} x_i$ for any $i = 1, 2, \ldots, n$ named inverse pairs. A word is assumed to be reduced if it holds no inverse pair sub-word. W_1 is not reduced word because it contain $c^{-1}c$ but W_2 and W_3 is reduced word.

Next, it's defined new symbol, $(X^{\pm 1})^*$, that is the set of words. It's defined function from $(X^{\pm 1})^* \times (X^{\pm 1})^*$ to $(X^{\pm 1})^*$ by writing the words that are to be multiplied. We have to introduce an equivalence relation in $(X^{\pm 1})^*$, to obtain a group from $(X^{\pm 1})^*$.

Two word are said to be equivalent if one of the words that can be attained from the former by a finite succession of insertion and deletion of expressions $x_i x_i^{-1}$ or $x_i^{-1} x_i$ within the word; $x_i \in X$. It will be convenient to refer to the pair $x_i x_i^{-1}$ or $x_i^{-1} x_i$ together; therefore when $x_j = x_i^{-1}$ is in X^{-1}, defined $x_j^{-1} = (x_i^{-1})^{-1} = x_i$. This is confirmed in the following definition:

Definition 3.1 [7]. Two words V and W are equivalent if one of the word can be achieved from the other by finite a finite succession of insertion and deletion of expressions of the form $x_i^{\varepsilon_i} x_i^{-\varepsilon_i}$, $x_i \in X$, $\varepsilon_i = \pm 1$. If V and W equivalent, then symbolized by $V \sim W$.

Clear that relation "$\sim$" in Definition 3.1 is equivalence relation in $(X^{\pm 1})^*$. Let $[W]$ is equivalence class of word W, then set of equvalence classes $\{[W]; W \in (X^{\pm 1})^*\}$ with binary operation $[V][W] = [VW]$ for every $V, W \in (X^{\pm 1})^*$ thereby becomes a group. This group is called free group on X, and symbolyzed by $F(X)$. The identity element of $F(X)$ is empty word and inverse of $[W]$ is $[W^{-1}]$, written $[W]^{-1} = [W^{-1}]$, for every $W \in (X^{\pm 1})^*$. We have the next theorem.

Theorem 3.1 [7]. *Let X be a set and $(X^{\pm 1})^*$ is set of words built from $X^{\pm 1} = X \cup X^{-1}$, then product operation definition $(X^{\pm 1})^*$ descent in a well-defined fashion to the set $F(X)$ of equivalence classes of member of $(X^{\pm 1})^*$, and $F(X)$ thereby becomes a group.*

Suppose that G is a group. If on G is defined a set X as in Theorem 3.1 and there is one or some words in $(X^{\pm 1})^*$ which is the same as an empty word called a relator set and symbolized by R (R is a set of words on $(X^{\pm 1})^*$ which is the same as an empty word), then the system $\langle X \mid R \rangle$ is referred to a presentation of G, and usually written by $P = \langle X \mid R \rangle$, where X is set of generators and R is set of relations.

In this paper the process of determining the generator and relator for the group G is based on the characteristics of the elements in G. Noted that based on the Cayley table[ii] of G, it's found that there are several types of elements in G, that is:

The element that $a^4 = e$: A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{17}, A_{18}, A_{25}, A_{26}. Based on these types of element, it's concluded that there are some elements with $a^2 = a^4 = e$ will represent several elements, so that the element with this type is one of the generators of this presentation. Noted that, if the first assumption is taken that the generator of this presentation is two, then this assumption

[ii] Cayley table of $\mathcal{G}$

×	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28	A29	A30	A31	A32
A1	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26	A27	A28	A29	A30	A31	A32
A2	A2	A1	A4	A3	A6	A5	A8	A7	A10	A9	A12	A11	A14	A13	A16	A15	A18	A17	A20	A19	A22	A21	A24	A23	A26	A25	A28	A27	A30	A29	A32	A31
A3	A3	A4	A2	A1	A7	A8	A6	A5	A11	A12	A10	A9	A15	A16	A14	A13	A19	A20	A18	A17	A23	A24	A22	A21	A27	A28	A26	A25	A31	A32	A30	A29
A4	A4	A3	A1	A2	A8	A7	A5	A6	A12	A11	A9	A10	A16	A15	A13	A14	A20	A19	A17	A18	A24	A23	A21	A22	A28	A27	A25	A26	A32	A31	A29	A30
A5	A5	A6	A8	A7	A2	A1	A3	A4	A13	A14	A16	A15	A10	A9	A11	A12	A21	A22	A24	A23	A26	A17	A19	A20	A29	A30	A32	A31	A26	A25	A27	A28
A6	A6	A5	A7	A8	A1	A2	A4	A3	A14	A13	A15	A16	A9	A10	A12	A11	A22	A21	A23	A24	A17	A18	A20	A19	A30	A29	A31	A32	A25	A26	A28	A27
A7	A7	A8	A5	A6	A4	A3	A2	A1	A15	A16	A13	A14	A12	A11	A10	A9	A23	A24	A21	A22	A20	A19	A18	A17	A31	A32	A29	A30	A28	A27	A26	A25
A8	A8	A7	A6	A5	A3	A4	A1	A2	A16	A15	A14	A13	A11	A12	A9	A10	A24	A23	A22	A21	A19	A20	A17	A18	A32	A31	A30	A29	A27	A28	A25	A26
A9	A9	A10	A11	A12	A13	A14	A15	A16	A2	A1	A4	A3	A6	A5	A8	A7	A25	A26	A27	A28	A29	A30	A31	A32	A18	A17	A20	A19	A22	A21	A24	A23
A10	A10	A9	A12	A11	A14	A13	A16	A15	A1	A2	A3	A4	A5	A6	A7	A8	A26	A25	A28	A27	A30	A29	A32	A31	A17	A18	A19	A20	A21	A22	A23	A24
A11	A11	A12	A10	A9	A15	A16	A14	A13	A4	A3	A1	A2	A8	A7	A5	A6	A27	A28	A26	A25	A31	A32	A30	A29	A20	A19	A17	A18	A24	A23	A21	A22
A12	A12	A11	A9	A10	A16	A15	A13	A14	A3	A4	A2	A1	A7	A8	A6	A5	A28	A27	A25	A26	A32	A31	A29	A30	A19	A20	A18	A17	A23	A24	A22	A21
A13	A13	A14	A16	A15	A10	A9	A11	A12	A6	A5	A7	A8	A1	A2	A4	A3	A29	A30	A32	A31	A26	A25	A27	A28	A22	A21	A23	A24	A17	A18	A20	A19
A14	A14	A13	A15	A16	A9	A10	A12	A11	A5	A6	A8	A7	A2	A1	A3	A4	A30	A29	A31	A32	A25	A26	A28	A27	A21	A22	A24	A23	A18	A17	A19	A20
A15	A15	A16	A13	A14	A12	A11	A10	A9	A8	A7	A6	A5	A3	A4	A1	A2	A31	A32	A29	A30	A28	A27	A26	A25	A24	A23	A22	A21	A19	A20	A17	A18
A16	A16	A15	A14	A13	A11	A12	A9	A10	A7	A8	A5	A6	A4	A3	A2	A1	A32	A31	A30	A29	A27	A28	A25	A26	A23	A24	A21	A22	A20	A19	A18	A17
A17	A17	A18	A19	A20	A21	A22	A23	A24	A26	A25	A28	A27	A30	A29	A32	A31	A2	A1	A4	A3	A6	A5	A8	A7	A9	A10	A11	A12	A13	A14	A15	A16
A18	A18	A17	A20	A19	A22	A21	A24	A23	A25	A26	A27	A28	A29	A30	A31	A32	A1	A2	A3	A4	A5	A6	A7	A8	A10	A9	A12	A11	A14	A13	A16	A15
A19	A19	A20	A18	A17	A23	A24	A22	A21	A28	A27	A25	A26	A32	A31	A29	A30	A4	A3	A1	A2	A8	A7	A5	A6	A11	A12	A10	A9	A15	A16	A14	A13
A20	A20	A19	A17	A18	A24	A23	A21	A22	A27	A28	A26	A25	A31	A32	A30	A29	A3	A4	A2	A1	A7	A8	A6	A5	A12	A11	A9	A10	A16	A15	A13	A14
A21	A21	A22	A24	A23	A18	A17	A19	A20	A30	A29	A31	A32	A25	A26	A28	A27	A6	A5	A7	A8	A1	A2	A4	A3	A13	A14	A16	A15	A10	A9	A11	A12
A22	A22	A21	A23	A24	A17	A18	A20	A19	A29	A30	A32	A31	A26	A25	A27	A28	A5	A6	A8	A7	A2	A1	A3	A4	A14	A13	A15	A16	A9	A10	A12	A11
A23	A23	A24	A21	A22	A20	A19	A18	A17	A32	A31	A30	A29	A27	A28	A25	A26	A8	A7	A6	A5	A3	A4	A1	A2	A15	A16	A13	A14	A12	A11	A10	A9
A24	A24	A23	A22	A21	A19	A20	A17	A18	A31	A32	A29	A30	A28	A27	A26	A25	A7	A8	A5	A6	A4	A3	A2	A1	A16	A15	A14	A13	A11	A12	A9	A10
A25	A25	A26	A27	A28	A29	A30	A31	A32	A17	A18	A19	A20	A21	A22	A23	A24	A10	A9	A12	A11	A14	A13	A16	A15	A2	A1	A4	A3	A6	A5	A8	A7
A26	A26	A25	A28	A27	A30	A29	A32	A31	A18	A17	A20	A19	A22	A21	A24	A23	A9	A10	A11	A12	A13	A14	A15	A16	A1	A2	A3	A4	A5	A6	A7	A8
A27	A27	A28	A26	A25	A31	A32	A30	A29	A19	A20	A18	A17	A23	A24	A22	A21	A12	A11	A9	A10	A16	A15	A13	A14	A4	A3	A1	A2	A8	A7	A5	A6
A28	A28	A27	A25	A26	A32	A31	A29	A30	A20	A19	A17	A18	A24	A23	A21	A22	A11	A12	A10	A9	A15	A16	A14	A13	A3	A4	A2	A1	A7	A8	A6	A5
A29	A29	A30	A32	A31	A26	A25	A27	A28	A21	A22	A24	A23	A18	A17	A19	A20	A14	A13	A15	A16	A9	A10	A12	A11	A6	A5	A7	A8	A1	A2	A4	A3
A30	A30	A29	A31	A32	A25	A26	A28	A27	A22	A21	A23	A24	A17	A18	A20	A19	A13	A14	A16	A15	A10	A9	A11	A12	A5	A6	A8	A7	A2	A1	A3	A4
A31	A31	A32	A29	A30	A28	A27	A26	A25	A23	A24	A21	A22	A20	A19	A18	A17	A16	A15	A14	A13	A11	A12	A9	A10	A8	A7	A6	A5	A3	A4	A1	A2
A32	A32	A31	A30	A29	A27	A28	A25	A26	A24	A23	A22	A21	A19	A20	A17	A18	A15	A16	A13	A14	A12	A11	A10	A9	A7	A8	A5	A6	A4	A3	A2	A1

is not right. Furthermore, from Cayley table of G, we have a^4 are A_9 and A_{10} and b^2 are A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, and A_{16}. There are $C_2^5 = 10$ possible pairs that can be made.

Based on the information and pattern in Cayley table, it's concluded that this presentation was built by more than two generators, that is, it has 4 generators, that is a, b, c, and d. The relations that can represent all elements are $a^4 = b^4 = c^2 = d^2$, $a^2 = b^2$, $ab = ba^{-1}$, $acd = dca^{-1}$ and $bcd = dcb^{-1}$. Thus, presentation of group G, is

$$P = \langle a, b, c, d \mid a^4 = b^4 = c^2 = d^2, a^2 = b^2, ab = ba^{-1}, acd = dca^{-1}, bcd = dcb^{-1} \rangle$$

So, the elements of G are

$$e, a, b, c, d, a^3, b^3, c^2, ab, a^2b, a^3b, ac, a^2c, a^3c, ad, a^2d, a^3d, bc, a^2bc, a^3bc, bd, a^2bd, cd, a^2cd, a^3cd, abc, abd, acd, bcd, abcd \tag{1}$$

We conclude this paper with the next theorem.

Theorem 3.2. *Group presentation of G, is*

$$P = \langle a, b, c, d \mid a^4 = b^4 = c^2 = d^2, a^2 = b^2, ab = ba^{-1}, acd = dca^{-1}, bcd = dcb^{-1} \rangle$$

Proof. Based on (1), it's sufficient to prove this theorem that all elements in G can be represented by (1).

References

1. Yanita, Y., Helmi, M.R. and Zakiya, A.M., **2019**. Solvability group from Kronecker product on the representation of quaternion group. *Asian Journal of Scientific Research, 12*, pp.293–297.
2. Galian, J.A., **2010**. *Contemporary Abstract Algebra*, 7th edn., USA, Brook/Cole Cengange Learning.
3. Kurosh, A.G., **2003**. *Theory of Group*. USA, American Mathematical Society. Vol. 2.
4. Rotman, J.J., **1999**. *An Introduction to the Theory of Group*, 4th edn., New York, Springer-Verlag.
5. Wehrfritz, B.A.F., **2009**. *Group and Ring Theoritic Properties of Polycyclic Groups.* New York, Springer London Dordrecht Heidelberg.
6. Johnson, D.L., **1997**. *Presentations of Group*, 2nd edn., United Kingdom, Cambridge University,
7. Knapp, A.W., **2006**. *Basic Algebra*. New York, USA, Birkhäuser, Boston, Springer Science+Business Media LLC.

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