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A MODIFIED ECONOMIC PRODUCTION QUANTITY (EPQ) WITH SYNCHRONIZING DISCRETE AND CONTINUOUS DEMAND UNDER FINITE HORIZON PERIOD AND LIMITED CAPACITY OF STORAGE

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ABSTRACT

The most popular inventory model to determine production lot size is the Economic Production Quantity (EPQ) model. It aids enterprises on how to minimize the total of production costs by reducing the inventory cost. However, the three main parameters in EPQ model, demand, set up cost, and holding cost, are not sufficient enough to solve current inventory issues. When an enterprise has two types of demand, continuous and discrete demands, the basic EPQ model would be no longer useful. Continuous demand comes from customers who want their demand to be fulfilled every time per unit time, while the fulfilment of discrete demand is at a fixed interval of time. Literature review is conducted to observe other formulations of EPQ model. As literature dealing with this problem cannot be found, this study aims to develop an EPQ model considering the two types of demand simultaneously. Therefore, this research proposes a modified EPQ model considering both continuous and discrete demands under finite horizon period. To find the solution of the model, three solution approaches were developed: (1) procedure approach, (2) algorithm approach, and (3) simultaneous approach. A numerical example is used to demonstrate the model. The solutions of the numerical example obtained using the three solution approaches are discussed.
Key words: Economic Production Quantity, continuous demand, discrete demand

1. INTRODUCTION

Production planning is the process to implement strategies and aims of a company into production activity. One of production planning steps is determining production lot size. Famous production lot size model used in many companies is Classic Economic Production Quantity (EPQ) model. Ballou (1992) mentioned that there are three basic parameters in EPQ model. They are demand, setup cost and holding cost. But then Kostic (2007) argued that the three parameters are just not enough to face issues in business. Many researchers have developed the model to anticipate real situation.

Development of EPQ model widely includes multiple products, backorder policy, deteriorating items, rework and scrap in production. Classic EPQ model assumes that inventory fulfils the demand every time constantly. This assumption sometimes is not applicable in the real system so that the model cannot be applied directly to the system. One of them is where the company fulfil two types of demands that are discrete and continuous demands. Discrete demand is demand which is fulfilled in certain period interval. When the company also have the discrete demand, then a part of its production will be stored longer than when the demand is continuous so that holding cost will increase.

2. LITERATURE REVIEW

Classic EPQ model have developed widely because the model does not consider any aspects in the real system so that the application of the model is limited in real system (Pasandideh *et al*, 2010).

Development of EPQ model includes many aspects such as considering multiple products (Bera *et al*, 2009; Mandal dan Roy, 2006; Rezaei dan Davoodi, 2008; Taleizadeh, 2010), backorder policy (Pentico dan Drake, 2008; Chung dan Cardenas-Barron, 2012; Li *et* al, 2008; Pentico *et al,* 2011; Hseish dan Dye, 2012), deteriorating products (Panda *et al* 2007; Inderfurth *et al*, 2005; Teng dan Chang, 2005; Chung *et al*,

2011; Mahata, 2012) and rework and scrap in production (Mlkdashi, 2012; Chiu *et al*, 2010; Eroglu *et al* 2008; El-Kassar, 2008).
Also, one assumption which is ignored in EPQ model is that demand is not constant (Pasandideh dan Niaki, 2008) and not delivered or fulfilled every time (continuous) (Chiu *et al,* 2014). Constant or continuous demand in EPQ model currently is not realistic and not applicable in supply chain system (Wu *et al*, 2014). The company sometimes fulfil the demand in certain period interval (discrete) because of many considerations that are transportation mode, delivery distance, and operational needs of the company.
Chiu *et al* (2009) proposed EPQ model for discrete demand and rework and scrap in production. Delivery of products cannot be performed continuously because the company have to produce and rework the products until the number of products delivered is met. Frequency of the delivery is *n* in one production cycle. In 2012, Chiu *et al* proposed EPQ model by decreasing holding cost with changing the frequency of delivery to n+1 times in one production cycle.
Wu *et al* (2014) developed the model in Chiu *et al* (2012). In this research, Wu improve delivery policy with adding the number of delivery when production quantity is enough to deliver.
Other research, Taleizadeh *et al* (2015) formulated EPQ model with discrete demand to determine the price of product, production lot size and optimal frequency of delivery with considering rework in production. Chiu *et al* (2014) also proposed EPQ model with discrete demand by synchronizing scrap and rework. But, Chiu *et al* assumed that a part of products cannot be reworked and become scrap.
Based on literature review, the research about EPQ model considering both discrete and continuous demand simultaneously in a model is still far from the literature.
So that, this research is to develop EPQ model considering both discrete and continuous demand to be applicable in the real system.

3. MODEL FORMULATION

3.1 Assumption and Notation

In this research, EPQ model is developed with considering both discrete and continuous demand so that inventory behaviour can be illustrated in Figure 2.

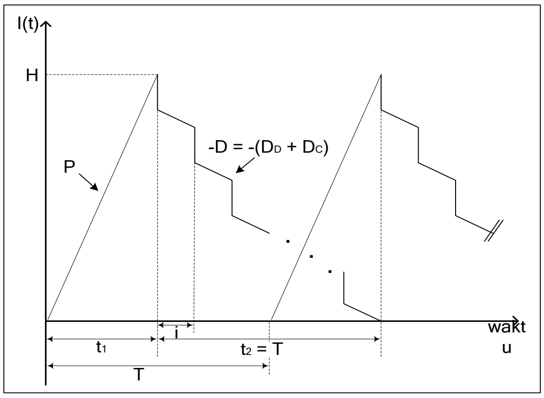

Figure 2**.** Inventory behaviour of EPQ model with two types of demand

Assumption:
1. Delivery interval is constant
2. All costs are constant
3. Price is constant and no discounts
4. No breakdown during production
5. No backorder and rework
6. No deterioration of machines and equipment during production
7. Production and delivery capacity are constant
8. No imperfect products
9. No safety stock
10. During production, no demand. Demand in the cycle is fulfilled by production in the previous cycle.
11. Single item product.

Notation:
<u>Decision variable</u>
T : production cycle (period)

<u>Parameters</u>
H : Maximum production inventory level (units)
t_1 : production cycle time (time)
t_2 : delivery time for one production cycle (time)
Q : production lot size (units)
I(t) : Inventory level at *t* (units)
c_p : production cost (Rp/unit)

c_s : Setup cost (Rp)
c_f : fixed delivery cost (Rp)
c_d : variable delivery cost (Rp/units)
h : holding cost (Rp/units.time)
h_1 : holding cost at buyer (Rp/units.time)
D : total demand (units/time)
D_D : discrete demand (units/ time)
D_C : continuous demand (units/ time
p : production capacity (units/ time)
n : delivery frequency of discrete demand in one cycle (integer number)

TC(T,n) : Total relevant cost per cycle (Rp)
E[TCU(T,n)] : Expected total relevant cost in one production cycle (Rp)

3.2. Mathematical Formulation

To calculate total production cost per cycle, production cost per unit, c_p is multiplied by production lot size, *Q*. Formulation for the total production cost per cycle is derived as follows

$$\textbf{\textit{Total production cost per cycle}} = \boldsymbol{c_p.Q} \quad (1)$$

Frequency of stock replenishment in one year is that demand of the product, *D* is divided by production lot size, *Q*, that is $1/T = D/Q$. Therefore, $Q = TD$, so that production cost in one cycle becomes

$$Total\ production\ cost\ per\ cycle = c_p.T.D \quad (2)$$

Then, setup cost is as follows.

$$\textbf{\textit{Setup cost per cycle}} = \boldsymbol{c_s} \quad (3)$$

Delivery cost of the products can be devided into two that are fixed cost per delivery, c_f per each delivery *n* for the discrete demand and variabel cost per delivery , c_d per unit of the products for both types of demand. Total delivery cost in one cycle is;

$$Total\ delivery\ cost\ per\ cycle = (n.c_f) + (c_d.T.D) \quad (4)$$

Discrete demand is delivered *n* times in each cycle with considering delivery interval *i*, whereas continuous demand is delivered every time during t_2. Figure 2 shows that:

$$\boldsymbol{t_1 = \frac{Q}{P} = \frac{TD}{P}} \quad (5)$$
$$\boldsymbol{H = TD} \quad (6)$$
maka,
$$\boldsymbol{H_D = Q = TD_D} \quad (7)$$
$$\boldsymbol{H_C = Q = TD_C} \quad (8)$$

The computation of holding costs is divided by two types that are holding cost for continuous demand and discrete demand.

3.2.1. Holding cost for continuous demand

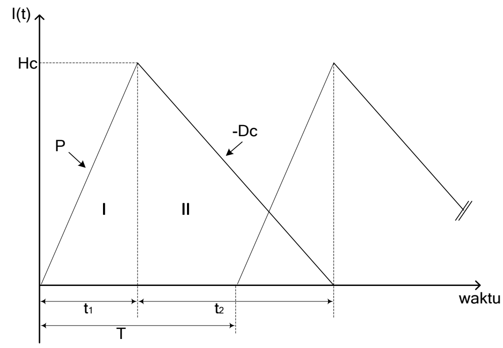

Figure 3. Inventory level for continuous demand of EPQ model

Average inventory in part I:

$$= \frac{1}{2}.t_1.Q_C$$
$$= \frac{1}{2}.\left(\frac{TD}{P}\right)\left(T.D_C\right)$$
$$= \frac{1}{2P}.T^2.D.D_C \quad (9)$$

Average inventory in part II:

$$= \frac{1}{2}.T.Q_C$$
$$= \frac{1}{2}.T.(T.D_C)$$
$$= \frac{1}{2}.T^2.D_C \quad (10)$$

With holding cost per unit time, h, so that holding cost for continuous demand in one cycle is:

$$Holding\ cost\ for\ continuous\ demand\ per\ cycle = h.\left[\left(\frac{1}{2P}.T^2.D.D_C\right) + \left(\frac{1}{2}.T^2.D_C\right)\right] \quad (11)$$

3.2.2. Holding cost for discrete demand

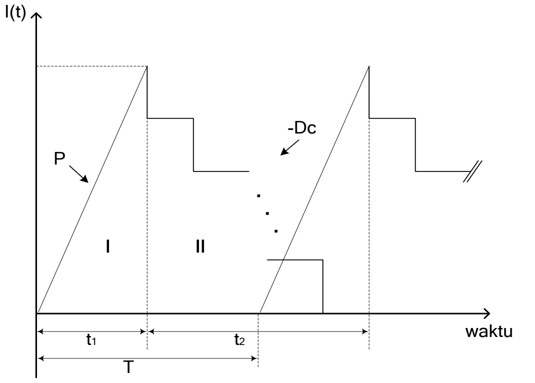

Figure 4. Inventory level for continuous demand of EPQ model

Average inventory in part I:

$$= \frac{1}{2}.t_1.Q_D$$
$$= \frac{1}{2}.\left(\frac{TD}{P}\right)\left(T.D_D\right)$$
$$= \frac{1}{2P}.T^2.D.D_D \quad (12)$$

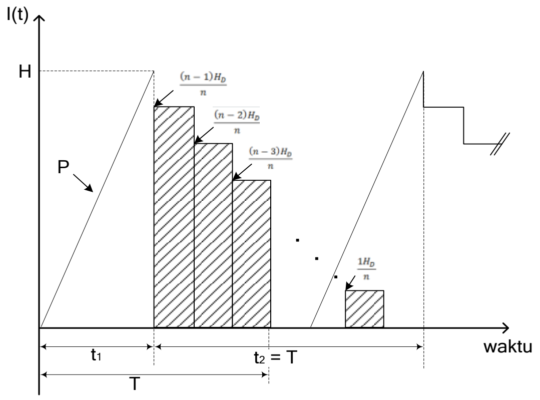

Figure 5. Inventory level for discrete demand during t_2 in EPQ model (Chiu *et al*, 2009)

In part II, products are delivered *n* times in each interval of *T/n* with quantity of the products delivered is equal to maximum inventory, $H_D = Q_D$. Figure 4 shows that inventory level during t_2 to fulfil the discrete demand. Equation of holding cost for discrete demand in part II referred to Chiu *et al* (2009)

Average inventory in part II:

$$= \left(\frac{(n-1)H_D}{n}\frac{T}{n}\right)+\left(\frac{(n-2)H_D}{n}.\frac{T}{n}\right)+\left(\frac{(n-3)H_D}{n}.\frac{T}{n}\right)+...+\left(\frac{1.H_D}{n}.\frac{T}{n}\right)$$
$$= \frac{H_D T}{n^2}\left[(n-1)+(n-2)+(n-3)+...+1\right]$$
$$= \frac{H_D.T}{n^2}.\frac{n(n-1)}{2}$$
$$= \frac{(n-1).H_D T}{2n}$$

With considering that H_d=Q_d, so average inventory in part II is:

$$= \left(\frac{n-1}{2n}\right)T.Q_D$$
$$= \left(\frac{n-1}{2n}\right)T.(T.D_D)$$
$$= \left(\frac{n-1}{2n}\right)T.^2 D_D \quad (13)$$

Then, holding cost for discrete demand in one cycle is:

$$Holding\ cost\ for\ discrete\ demand\ per\ cycle$$
$$= h.\left[\left(\frac{1}{2P}.T^2.D.D_D\right)+\left(\left(\frac{n-1}{2n}\right).T^2.D_D\right)\right] \quad (14)$$

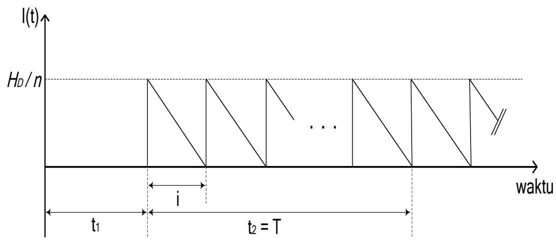

Figure 6. Inventory level for discrete demand of buyer during t_2 in EPQ model

Beside the holding cost in warehouse, EPQ model considers holding cost per unit of products of buyers. When the products are received by buyers as the number of HD/n, use of products is not simultaneous but gradual.

Discrete demand H_D is delivered *n* times in one cycle in fixed time interval *i*, so that

$$i = \frac{T}{n} \quad (15)$$

Based on figure 6, inventory quantity in each delivery is

$$= \frac{1}{2}.\frac{H_D}{n}.\frac{T}{n} \quad (16)$$

With delivery frequency per cycle is *n* so the inventory quantity in one cycle is

$$= \left(\frac{1}{2}.\frac{H_D}{n}.\frac{T}{n}\right)n$$

$$= \frac{H_D.T}{2n} \quad (17)$$

Substituting equation (7), so that inventory quantity in one cycle becomes

$$= \frac{T^2.D_D}{2n} \quad (18)$$

Then, holding cost for discrete demand of the buyers in one cycle is

$$Holding\ cost\ for\ the\ buyers\ per\ cycle = \frac{h_1.T^2.D_D}{2n} \quad (19)$$

In this paper, capacity of warehouse/storage is limited, so that

$$Q = T.D \leq C \quad (20)$$

where

C = Capacity of warehouse/ storage

and also, production and inventory planning is for a finite horizon period. Then,

$$\frac{Y}{T} = M \quad (21)$$

where

Y = the finite horizon period (year)

M = an integer number

3.3. Objective Function

Total cost per cycle, *TC(T,n)* consists of production cost, setup cost, holding cost of producer and buyers and fixed and variable delivery costs. Then, *TC(T,n)* is the sum of equation (1), (3), (4), (11), (14), and (19) so that

$$TC(T,n) = c_pTD + c_s + \left(nc_f + c_dTD\right) + \left(\frac{hT^2DD_C}{2P} + \frac{hT^2D_C}{2}\right) + \left[\frac{hT^2DD_D}{2P} + \frac{(n-1)T^2D_D}{2n}\right] + \frac{h_1T^2D_D}{2n} \quad (22)$$

Objective function is average total cost per period, *E [TCU(T,n)]*. *E [TCU(T,n)]* is formulated as follows

$$E[TCU(T)] = \frac{E[TC(T,n)]}{T} \quad (23)$$

$$= c_pD + \frac{c_s}{T} + n\frac{c_f}{T} + c_dD + \frac{hTD}{2P}(D_C + D_D) + \frac{hT}{2}(D_C + D_D) + \frac{TD_D}{2n}(h_1 - h)$$

3.4. Optimal Solution Procedure

Optimal Inventory cycle time is derived with minimizing $E[TCU(T)]$. Differentiation of *E[TCU(T)]* to *T* is as follows:

$$\frac{d\,E[TCU(T)]}{dT} = -\frac{c_s}{T^2} - n\frac{c_f}{T^2} + c_dD + \frac{hD^2}{2P} + \frac{hD}{2} + \frac{D_D(h_1-h)}{2n} \quad (24)$$

Furthermore, set equation (24) equal to zero.

$$\frac{d\,E[TCU(T)]}{dT} = 0 = -\frac{c_s}{T^2} - n\frac{c_f}{T^2} + c_dD + \frac{hD^2}{2P} + \frac{hD}{2} + \frac{D_D(h_1-h)}{2n} \quad (25)$$

After arranging both sides, we obtain

$$\frac{c_s}{T^2} + n\frac{c_f}{T^2} = c_dD + \frac{hD^2}{2P} + \frac{hD}{2} + \frac{D_D(h_1-h)}{2n} \quad (26)$$

so that

$$T = \sqrt{\frac{2(C_S + nC_f)}{\frac{hD^2}{P} + hD + \frac{D_D(h_1-h)}{n}}} \quad (27)$$

Because there are some constraints of the model, the solution can be derived using mixed integer non-linear programming.

4. NUMERICAL EXAMPLE

Known production rate of plant X is 240 units per minute and the products are produced to fulfil total demand as amount of 80 million per year. Percentages of each continuous and discrete demand are 60% and 40%. Total working time in a day is 21 hours and number of working days in a year is 360 days. Other parameters considered in this numerical example are: c_s= Rp 20 juta; c_p = Rp1,540 per sheet; c_f = Rp 2.5 million per delivery; c_d = Rp100 per unit; h = Rp 440 per unit; Y = 5 years, h_1 = Rp 880 per unit, and C = 6,000,000 units. Using Software Lingo version 13.0, we obtain that optimal production cycle time, T^* = 10.83 days, n = 2, and average total costs in a period, *E[TCU(T)]* is Rp133.092.400.000.

5. CONCLUSION

This research proposed a model to find optimal solution of *Economic Production Quantity* (EPQ) with considering two types demand that continuous and discrete demand simultaneously. The model provides new paradigm in solving production planning regarding to two types of demand.
There are still some limitations of the model. Next research focus on considering many aspects such as defect products, multi item products, and rework in production.

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