# Asian Journal of Scientific Research

ISSN 1992-1454

OPEN ACCESS

Asian Journal of Scientific Research

ISSN 1992-1454
DOI: 10.3923/ajsr.2018.409.414

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## Research Article
# A New Variant of Chebyshev-Halley's Method Without Second Derivative with Convergence of Optimal Order

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## Abstract
**Background and Objective:** The Chebyshev-Halley is an third order iterative method that be used to find the roots of a nonlinear equation. This study is presented a new variant of Chebyshev-Halley's method without second derivative with two parameters. **Methodology:** In order to avoid the second derivative, it is approximated by using an equality of two methods, namely, use of a circle of curvature that has the same tangent line and to equate to the Potra-Ptak's method. **Results:** The results show that the method requires two evaluation of functions and one its first derivative per iteration with the efficiency index equal to $4^{1/3} \approx 1.5874$. The convergence analysis shows that the proposed method has the fourth-order convergence for $\theta = 1$ and $\beta = 1$ and requires three evaluation of functions per iteration. **Conclusion:** The final results show that the proposed methods has better performance as compared some other kind of methods. A numerical simulation is presented to show the performance of the proposed method by using several functions.

Key words: Chebyshev-Halley's method, convergence, optimal order, nonlinear equation, Newton's method

Received: February 10, 2018 Accepted: April 26, 2018 Published: June 15, 2018

Citation: Wartono, M. Soleh, I. Suryani, Zulakmal and Muhafzan, 2018. A new variant of Chebyshev-Halley's method without second derivative with convergence of optimal order. Asian J. Sci. Res., 11: 409-414.

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Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

## INTRODUCTION

The problems of determining the roots of a nonlinear equation constitutes one of the very important problem in numerical analysis. It is well-known that the following Newton's method, i.e:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

constitutes as classical iterative method that can be used to find a simple root of a nonlinear equation $f(x) = 0$, where $f : D \subset \Re \rightarrow \Re$ is a scalar function. As Ostrowski[1] is stated that this method is quadratically convergent with efficiency indexes equal to $\sqrt[2]{2} \approx 1.4142$.

In order to improve the local order of convergence, many modification of the method have been proposed. A family of iterative method with third order-convergence has been reported by Amat *et al.*[2] and Hernandez and Salanova[3] as follows:

$$x_{n+1} = x_n - \left(1 + \frac{L_f(x_n)}{2(1-\beta L_f(x_n))}\right)\frac{f(x_n)}{f'(x_n)} \tag{2}$$

Where:

$$L_f(x_n) = \frac{f''(x_n)f(x_n)}{f'(x_n)^2} \tag{3}$$

The Eq. 2 is known as Chebyshev-Halley's method for some β. In particularly, the Eq. 2 becomes the Chebyshev's method if $\beta = 0$, Halley's method if $\beta = 1/2$ and super Halley's method if $\beta = 1$, according to Gutierrez and Hernandez[4].

Note that the Eq. 2 requires a second derivative of f. It is evident that the Eq. 2 can not be used in the cases in which the second derivative of f is not exist. Recently, some modification of Eq. 2 have been studied to avoid the second derivative by using several approximation such as Taylor's series expansion[5-8], finite different quotient[9-11], cubic polynomial[12], quadratic function[13], linear combination[14] and hyperbola[15].

Motivated by the recent study, in this paper is presented a new variant of the classical Chebyshev-Halley's method that contain two real parameters using a new approximation to avoid the second derivative o f f in Eq. 2. It is shown that for $\theta = 1$, this new method constitutes a generalization of several previous methods that be proposed in Ostrowski[1], Chun[16], Potra and Ptak[17] and Sharma[18]. In the end of this study, a numerical simulation is presented for comparing several methods.

## NEW METHOD

To derive this method, let us consider the Chebyshev-Halley's method in Eq. 2 in the following form:

$$x_{n+1} = x_n - \left(1 + \frac{f(x_n)f''(x_n)}{2(f'(x_n)^2 - \beta f''(x_n)f(x_n))}\right)\frac{f(x_n)}{f'(x_n)} \tag{4}$$

The iteration scheme of Eq. 4 has the third order of convergence and contains a second derivative function. In order to avoid the second derivative, $f''(x_n)$ is approximated by using an equality of two methods.

To derive an approximation for $f''(x_n)$ in Eq. 4, firstly use a circle of curvature that has a same tangent line at $(x_n, y_n)$ of the curve $y = f(x)$ that given by:

$$\left(x - x_n + \frac{y'_n(1+y'^2_n)^2}{y''_n}\right)^2 + \left(y - y_n + \frac{1+y'^2_n}{y''_n}\right)^2 = \frac{(1+y'^2_n)^3}{y''^2_n} \tag{5}$$

The circle of curvature in Eq. 5 that throughout at intersection at point $(x_{n+1}, 0)$ is:

$$(x_{n+1} - x_n)^2 + \frac{2y'_n(1+y'^2_n)^2}{y''_n}(x_{n+1} - x_n) + y_n^2 + \frac{2y_n(1+y'^2_n)}{y''^2_n} = 0 \tag{6}$$

Let $f(x_n) = y_n$, $f'(x_n) = y'_n$ and $f''(x_n) = y''_n$, then (6) can be written as:

$$x_{n+1} = x_n - \frac{\left((x^*_{n+1} - x_n)^2 + f(x_n)^2\right)f''(x_n) + 2f(x_n)\left(1 + f'(x_n)^2\right)}{2f'(x_n)\left(1 + f'(x_n)^2\right)} \tag{7}$$

Where:

$$x^*_{n+1} = x_n - \theta\frac{f(x_n)}{f'(x_n)}, \theta \in \Re \tag{8}$$

By substituting (8) into (7), one have:

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)^2\left(1 + f'(x_n)^2\right) + 2f(x_n)^2 f''(x_n)\left(\theta + f(x_n)^2\right)}{2f'(x_n)^3\left(1 + f'(x_n)^2\right)} \tag{9}$$

Furthermore, consider the following Potra-Ptak's method[17]:

$$x^*_{n+1} = x_n - \frac{f(x_n) + f(y_n)}{f'(x_n)} \tag{10}$$

where $y_n$ is a Newton's method that defined by (8):

Based on Eq. 9 and 10, one get a new expression of $f''(x_n)$ that given by:

$$f''(x_n) = \frac{2f(y_n)f'(x_n)^2\left(1+f'(x_n)^2\right)}{f(x_n)^2\left(\theta^2+f'(x_n)^2\right)} \quad (11)$$

By subtituting Eq. 11 into Eq. 4, one obtained a new two-parameters family of Chebyshev-Halley's method which free of second derivative, that is:

$$x_{n+1} = x_n - \left(\frac{f(y_n)f'(x_n)^2\left(1+f'(x_n)^2\right)}{f(x_n)\left(\theta^2+f'(x_n)^2\right) - 2\beta f(y_n)\left(1+f'(x_n)^2\right)}\right)\frac{f(x_n)}{f'(x_n)} \quad (12)$$

The Eq. 12 is a variant of Chebyshev-Halley's method with two parameters θ and β that requires three evaluation of functions $f(x_n)$, $f'(x_n)$ and $f(y_n)$.

One can see that for $\theta = 1$ and $\beta \in \Re$, the family of Eq. 12 constitutes a generalization of Chebyshev-Halley's method. For $\beta \to \pm\infty$, one get the Newton's method as defined by Eq. 1. For $\beta = 0$, one get the Potra-Ptak's method[17]:

$$x_{n+1} = x_n - \left(1+\frac{f(y_n)}{f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}$$

For β = ½, one get the Newton-Steffensen's method[18]:

$$x_{n+1} = x_n - \left(1+\frac{f(y_n)}{f(x_n)-f(y_n)}\right)\frac{f(x_n)}{f'(x_n)}$$

For β =1, one get the Ostrowski's method[1]:

$$x_{n+1} = x_n - \left(1+\frac{f(y_n)}{f(x_n)-2f(y_n)}\right)\frac{f(x_n)}{f'(x_n)}$$

For β = -1/2, one get the Chun's method[16]:

$$x_{n+1} = x_n - \left(1+\frac{f(y_n)}{f(x_n)+f(y_n)}\right)\frac{f(x_n)}{f'(x_n)}$$

Furthermore, it will be shown that this new method has the fourth-order convergence.

**Theorem:** Let $\alpha \in D$ be a simple root of a differentiable function $f: D \subset \Re \to \Re$. If the initial value $x_0$ is sufficiently close to α, then the method defined by Eq. 12 has fourth order convergence for θ = 1 and β =1 with error:

$$e_{n+1} = c_2(c_2^2 - c_3)e_n^4 + O(e_n^5) \quad (13)$$

**Proof:** Let α is the root of nonlinear equation $f(x) = 0$. If $e_n = x_n - \alpha$ and $c_j = \frac{1}{j}\frac{f^{(j)}(\alpha)}{f'(\alpha)}$, then the expansion of Taylor's series for $f(x_n)$ and $f'(x_n)$ around α is given by:

$$f(x_n) = f'(\alpha)\left(e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + O(e_n^5)\right) \quad (14)$$

and:

$$f'(x_n) = f'(\alpha)\left(1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_4e_n^4 + O(e_n^5)\right) \quad (15)$$

because:

$$f'(x_n)^2 = f'(\alpha)^2\left(1 + 4c_2e_n + (4c_2^2+6c_3)e_n^2 + (12c_2c_3+8c_4)e_n^3 + \cdots + O(e_n^5)\right) \quad (16)$$

Using the Eq. 14 and 15, one get $\frac{f(x_n)}{f'(x_n)}$ as follows:

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + 2(c_2^2 - c_3)e_n^3 + \cdots + O(e_n^5) \quad (17)$$

Furthermore, using the Eq. 17 and $x_n = \alpha + e_n$, one get:

$$y_n = \alpha + (1-\theta)e_n + \theta c_2e_n^2 + 2\theta(c_3 - c_2^2)e_n^3 + \cdots + O(e_n^5) \quad (18)$$

and using the expansion of Taylor's series around α, $f(y_n)$ can be written as:

$$\begin{aligned} f(y_n) = f'(\alpha)\big((1-\theta)e_n &+ (\theta^2-\theta+1)c_2e_n^2 + 2\theta(c_3-c_2^2)e_n^3 \\ &+ ((5\theta^2-8\theta)c_2^3 - (4\theta^2+3\theta)c_2c_3 + 3\theta c_4)e_n^4 + O(e_n^5)\big) \end{aligned} \quad (19)$$

By using the Eq. 16 and 19, one get:

$$\begin{aligned} f(y_n)(1+f'(x_n)^2) = f'(\alpha)\Big((1-\theta)(1+f'(\alpha))e_n &+ \left((\theta^2-5\theta+5)f'(\alpha) + (1-\theta)^2\right)c_2e_n^2 \\ &+ \left(\left(2(\theta-2)^2f'(\alpha)^2 - 2\theta^2\right)c_2^2 + \left((6-4\theta)f'(\alpha)^2 + 2\theta\right)c_3\right)e_n^3 \\ &+ \Big(\left((\theta-2)^2f'(\alpha)^2 + 5\theta^2\right)c_2^3 + \left((2\theta^2-13\theta+18)f'(\alpha)^2 - (4\theta^2+3\theta)\right)c_2c_3 \\ &+ \left((-5\theta+8)^2f'(\alpha)^2 + 3\theta\right)c_4\Big)e_n^4 + O(e_n^5)\Big) \end{aligned} \quad (20)$$

$$\begin{aligned} f(x_n)(\theta+f'(x_n)^2) = f'(\alpha)\Big((\theta^2+f'(\alpha)^2)e_n &+ (\theta^2+5f'(\alpha)^2)c_2e_n^2 + (8c_2^2+(\theta^2+5f'(\alpha))c_3)e_n^3 \\ &+ \left(4f'(\alpha)^2c_2^3 + 22f'(\alpha)^2c_2c_3 + (\theta^2+9f'(\alpha)^2)c_4\right)e_n^4 + O(e_n^5)\Big) \end{aligned} \quad (21)$$

and:

$$2\beta f(y_n)(1+f'(x_n)^2) = 2\beta f'(\alpha)\Big((1-\theta)(1+f'(\alpha))e_n + \big((\theta^2-5\theta+5)f'(\alpha)+(1-\theta)^2\big)c_2e_n^2 \\ +\Big(2\big((\theta-2)^2 f'(\alpha)^2 - \theta^2\big)c_2^2 + 2\big((3-2\theta)f'(\alpha)^2+\theta\big)c_3\Big)e_n^3 \\ +\Big(\big((\theta-2)^2 f'(\alpha)^2+5\theta^2\big)c_2^3 + \big((2\theta^2-13\theta+18)f'(\alpha)^2-(4\theta^2+3\theta)\big)c_2c_3 \\ +\big((-5\theta+8)^2 f'(\alpha)^2+3\theta\big)c_4\Big)e_n^4 + O(e_n^5)\Big) \quad (22)$$

Furthermore, by using the Eq. 20-22 and $e_n = x_n - \alpha$, the Eq. 12 becomes:

$$e_{n+1} = \frac{(\theta-1)A_1}{\theta^2-1+A_1(1+2\beta(\theta-1))}e_n \\ + \frac{c_2(\theta-1)\big(4(\theta-1)A_1\beta^2+2(2\theta-\theta+(3A_1-2))A_1\beta-(A_0\theta^3-(2A_1-3)\theta^2+A_0^2(\theta+2)+5A_0)\big)}{\big(\theta^2-1+A_1(1+2\beta(\theta-1))\big)^2}e_n^2 \\ + \frac{1}{\big(\theta^2-1+A_1(1+2\beta(\theta-1))\big)^3}\Big[\Big(16\beta A_1^3(\theta-1)^3 \\ +\beta^2A_1^2(24\theta^4-8(6+A_1)\theta^3+48A_1\theta^2-24(3A_1-2)\theta+8(4A_1-5)) \\ +\beta(2A_1^2\theta^6-4A_6A_1\theta^5+2A_7\theta^4+8A_8\theta^3-8A_1A_9\theta^2+4A_{10}\theta-4A_{11}) \\ +(-2\theta^6-2A_1\theta^5+2(1-2A_{12})\theta^4+4A_{13}\theta^3-2A_{14}\theta^2-2A_{15}\theta+2A_{16})\Big)c_2^2 \\ +\Big(-16\beta^3A_1^3\theta(\theta-1)(\theta-2)+8\beta^2A_1^2(-3\theta^4+(6+A_1)\theta^3-6A_1\theta^2+4(3A_1-2)\theta-22(4A_1-5) \\ +2\beta A_1(-6\theta^5+(A_1+12)\theta^4-12A_1\theta^3+A_{17}\theta^2-6A_0(3A_1+1)\theta-22A_0A_{11}) \\ +(-2\theta^6-(A_1-6)\theta^5+3A_1\theta^4+4A_0(A_1+3)\theta^3-6A_0(A_1+1)\theta^2+A_0^2(5A_1+6)\theta \\ -A_0^2(5A_1+4)\Big)c_3\Big]+L+O(e_n^5) \quad (23)$$

Where:

$A_0 = f'(\alpha)^2$

$A_1 = 1+f'(\alpha)^2$

$A_2 = 4f'(\alpha)^2-1$

$A_3 = f'(\alpha)^6+22f'(\alpha)^4+f'(\alpha)^2+6$

$A_4 = -f'(\alpha)^6-4f'(\alpha)^4+7f'(\alpha)^2+2$

$A_5 = f'(\alpha)^2\,(7f'(\alpha)^4+27f'(\alpha)^2+4)$

$A_6 = f'(\alpha)^4\,(5f'(\alpha)^4+15f'(\alpha)^2+2)$

$A_7 = f'(\alpha)^2\,(2f'(\alpha)^4+7f'(\alpha)^2-2)$

$A_8 = 7f'(\alpha)^4+24f'(\alpha)^2+5$

Using the Eq. 23 and by taking θ = 1, one get:

$$e_{n+1} = -2(\beta-1)c_2^2e_n^3-((4\beta^2-14\beta+9)c_2^3+(8\beta-7)c_2c_3)e_n^4+O(e_n^5) \quad (24)$$

Because the Eq. 12 requires three evaluation of functions, it becomes an optimal iterative method when it has fourth order of convergence[19]. So, based on the Eq. 24, one can see that the order of convergence of Eq. 24 will increase quartically by taking β = 1 and it can be written as:

$$e_{n+1} = c_2(c_2^2-c_3)e_n^4+O(e_n^5) \quad (25)$$

The Eq. 25 has fourth-order convergence and it requires three evaluation of functions with an efficiency index equal to $4^{1/3} \approx 1.5874$.

## NUMERICAL EXAMPLE

In this section is presented a numerical example to illustrate efficiency of the proposed method by using several test functions. The zeros approximation α of the test functions was displayed 20th decimal places.

It is compared the performance of Eq. 12 both of β ≠ 1 (VCH3) and β = 1 (VCH4) with Newton's method (N2)[20], classical Chebyshev-Halley's method with β = 1/2 (CH3)[3,4], Potra-Ptak's method (PP3)[17]. All computations are performed by using Maple 13.0 with 850 digits floating point arithmetics for the following several test functions:

- $f_1(x) = xe^{-x} - 0.1$, α = 0.11183255915896296483
- $f_2(x) = e^x - 4x^2$, α = 4.30658472822069929833
- $f_3(x) = \cos(x) - 1$, α = 0,73908513321516064165
- $f_4(x) = (x-1)^3 - 1$, α = 2.00000000000000000000
- $f_5(x) = x^3 + 4x^2 - 10$, α = 1.36523003414096845760
- $f_6(x) = e^{-x^2+x+2} - \cos(x+1) + x^3 + 1$, α = -1.000000000 00000000000
- $f_7(x) = \sqrt{x} - x$, α = 1.00000000000000000000

Table 1 shows the number of iteration (IT) that satisfies stopping criteria:

$$|x_{n+1} - x_n| \le \varepsilon \quad (26)$$

where, e = $10^{-95}$ and all computation order of convergence (COC) in the parentheses by using the following formula:

$$\rho = \frac{\ln\left|\frac{(x_{n+1}-\alpha)}{(x_n-\alpha)}\right|}{\ln\left|\frac{(x_n-\alpha)}{(x_{n-1}-\alpha)}\right|} \quad (27)$$

Based on the Table 1, one can see that order of convergence of the proposed method is three for β ≠1 and four for β = 1.

The accuracy of the new method and several other methods by using the same total number of functional evaluation as comparison are presented at Table 2. Based on the Table 2, one can see that accuracy of the proposed method for β = 1 is better than other methods.

Table 1: Number of iteration (IT) and COC

| f (x) | $x_0$ | N2 | CH3 | PP3 | VCH3 | VCH4 |
|---|---|---|---|---|---|---|
| $f_1(x)$ | -0.2 | 8 (1.9999) | 5(3.0000) | 5 (3.0000) | 5 (2.9999) | 4 (3.9999) |
| | 0.3 | 8 (1.9999) | 5(3.0000) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |
| $f_2(x)$ | 4.0 | 8 (1.9999) | 6(3.0000) | 6 (3.0000) | 5 (2.9999) | 4 (3.9999) |
| | 4.5 | 7 (1.9999) | 5(3.0000) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |
| $f_3(x)$ | -0.5 | 8 (1.9999) | 5 (3.0000) | 6 (3.0000) | 6 (3.0000) | 5 (3.9998) |
| | 1.5 | 7 (1.9999) | 5 (3.0000) | 5 (3.0000) | 5 (2.9999) | 4 (3.9999) |
| $f_4$ (x) | 1.8 | 8 (1.9999) | 5 (3.0000) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |
| | 3.0 | 9 (1.9999) | 6 (3.0000) | 6 (2.9999) | 6 (3.0000) | 5 (3.9997) |
| $f_5$ (x) | 1.0 | 8 (1.9999) | 5 (3.0000) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |
| | 2.0 | 8 (1.9999) | 5 (3.0000) | 5 (2.9999) | 5 (2.9999) | 4 (3.9999) |
| $f_6$ (x) | -1.5 | 7 (1.9999) | 5 (3.0000) | 5 (2.9999) | 5 (2.9999) | 4 (3.9999) |
| | 0.0 | 7 (1.9999) | 6 (3.0000) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |
| $f_7$ (x) | 0.5 | 8 (1.9999) | 5 (3.0000) | 5 (3.0000) | 5 (2.9999) | 4 (3.9999) |
| | 1.5 | 7 (1.9999) | 5 (2.9999) | 5 (3.0000) | 5 (3.0000) | 4 (3.9999) |

Table 2: Absolute value of function $|f(x_n+1)|$ under same total number of functional evaluation (TNFE) with TNFE = 12

| f (x) | $x_0$ | N2 | CH3 | PP | VCH3 | VCH4 |
|---|---|---|---|---|---|---|
| $f_1$ (x) | -0.2 | 3.0851e-36 | 2.7750e-55 | 4.4605e-37 | 0.1272e-44 | 1.9110e-162 |
| | 0.3 | 1.0732e-42 | 3.5154e-66 | 2.2815e-37 | 0.9053e-53 | 1.0476e-192 |
| $f_2$ (x) | 4.0 | 5.0253e-33 | 2.1103e-53 | 4.2981e-23 | 5.5769e-41 | 3.5672e-157 |
| | 4.5 | 3.1919e-52 | 5.2464e-76 | 2.8677e-56 | 2.4261e-65 | 1.4627e-231 |
| $f_3$ (x) | -0.5 | 3.4884e-30 | 7.4037e-22 | 3.6137e-11 | 4.1176e-27 | 1.7116e-65 |
| | 1.5 | 3.7607e-64 | 1.1496e-51 | 6.5333e-72 | 3.5077e-79 | 4.9514e-201 |
| $f_4$ (x) | 1.8 | 2.8663e-41 | 1.7285e-60 | 3.9463e-35 | 42.3650e-51 | 6.5134e-180 |
| | 3.0 | 4.6449e-16 | 6.3909e-24 | 5.8202e-15 | 3.0961e-19 | 1.1038e-70 |
| $f_5$ (x) | 1.0 | 3.9823e-43 | 2.2349e-60 | 3.0006e-38 | 9.1052e-54 | 2.4510e-185 |
| | 2.0 | 1.2361e-37 | 4.6600e-52 | 4.0072e-39 | 7.8139e-47 | 3.6662e-161 |
| $f_6$ (x) | -1.5 | 5.7389e-66 | 1.5261e-43 | 1.7252e-67 | 5.1899e-91 | 1.3689e-166 |
| | 0.0 | 1.9261e-65 | 6.3918e-26 | 1.6494e-72 | 1.7252e-65 | 1.1346e-152 |
| $f_7$ (x) | 0.5 | 1.5492e-42 | 2.9666e-34 | 5.7100e-32 | 2.2096e-55 | 1.1552e-130 |
| | 1.5 | 1.0649e-66 | 5.4028e-17 | 1.3243e-84 | 2.4094e-84 | 5.2043e-227 |

## CONCLUSION

This research work have developed a new fourth-order convergence method for solving nonlinear equation that free from second derivative. The method requires two evaluation of functions and one its first derivative per iteration with the efficiency index equal to $4^{1/3} \approx 1.5874$. The numerical results show that the proposed method has better performance as compared with the other methods. Therefore, the results of this study provide a new contribution in computational science area.

## SIGNIFICANCE STATEMENT

This study discovers a new variant of Chebyshev-Halley's method as an alternative method to find the roots of the nonlinear equation. The results of this study can help the researchers in computational science and engineering area.

## REFERENCES


1. Ostrowski, A.M., 1973. Solution of Equations in Euclidean and Banach Space. Academic Press, New York.
2. Amat, S., S. Busquier and J.M. Gutierrez, 2003. Geometric constructions of iterative functions to solve nonlinear equations. J. Comput. Applied Math., 157: 197-205.
3. Hernandez, M.A. and M.A. Salanova, 1993. A family of Chebyshev-Halley type methods. Int. J. Comput. Math., 47: 59-63.
4. Gutierrez, J.M. and M.A. Hernandez, 1997. A family of Chebyshev-Halley type methods in banach spaces. Bull. Aust. Math. Soc., 55: 113-130.
5. Kanwar, V. and S.K. Tomar, 2007. Modified families of multi-point iterative methods for solving nonlinear equations. Numer. Algorithms, 44: 381-389.
6. Li, Y., P. Zhang and Y. Li, 2010. Some new variants of Chebyshev-Halley methods free from second derivative. Int. J. Nonlinear Sci., 9: 201-206.
7. Nedzhibov, G.H., V.I. Hasanov and M.G. Petkov, 2006. On some families of multi-point iterative methods for solving nonlinear equations. Numer. Algorithms, 42: 127-136.
8. Wartono, M. Soleh, I. Suryani and Muhafzan, 2016. Chebyshev-Halley's method without second derivative of eight-order convergence. Global J. Pure Applied Math., 12: 2987-2997.
9. Chun, C.B., 2007. Certain improvements of Chebyshev-Halley methods with accelerated fourth-order convergence. Applied Math. Comput., 189: 597-601.

10. Kou, J., Y. Li and X. Wang, 2007. Fourth-order iterative methods free from second derivative. Applied Math. Comput., 184: 880-885.
11. Rostami, M. and H. Esmaeili, 2014. A modification of Chebyshev-Halley method free from second derivatives for nonlinear equations. Caspian J. Math. Sci., 3: 133-140.
12. Chun, C., 2007. Some variants of Chebyshev-Halley methods free from second derivative. Applied Math. Comput., 191: 193-198.
13. Chun, C., 2007. Some second-derivative-free variants of Chebyshev-Halley methods. Applied Math. Comput., 191: 410-414.
14. Grau-Sanchez, M. and J.M. Gutierrez, 2010. Some variants of the Chebyshev-Halley family of methods with fifth order of convergence. Int. J. Comput. Math., 87: 818-833.
15. Xiaojian, Z., 2008. Modified Chebyshev-Halley methods free from second derivative. Applied Math. Comput., 203: 824-827.
16. Chun, C., 2008. A simply constructed third-order modifications of Newton's method. J. Comput. Applied Math., 219: 81-89.
17. Potra, F.A. and V. Ptak, 1984. Nondiscrete Induction and Iterative Processes. Vol. 103, Pitman Advanced Publisher, Boston, Massachusetts, USA., ISBN:9780273086277, Pages: 207.
18. Sharma, J.R., 2005. A composite third order Newton-Steffensen method for solving nonlinear equations. Applied Math. Comput., 169: 242-246.
19. Kung, H.T. and J.F. Traub, 1974. Optimal order of one-point and multipoint iteration. J. Assoc. Comput. Mach., 21: 643-651.
20. Traub, J.F., 1964. Iterative Methods for the Solution of Equations. Prentince-Hall Inc., New York.