Strategy Against Bird Flu Outbreak Within a Poultry Farm Based on Host-Virus Model

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Strategy Against Bird Flu Outbreak Within a Poultry Farm Based on Host-Virus Model

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ABSTRACT: Infection process of bird flu within a poultry farm is studied to propose strategies against outbreaks of bird-flu within a poultry farm. Mathematical models proposed in previous studies are extended to take spatial effects into consideration. Numerical results show that removal of infected birds is a crucial factor to avoid an outbreak in a poultry farm.

AMS (MOS) Subject Classification. 92D30, 92-08

1 INTRODUCTION

Bird flu or avian influenza is an epidemic disease among birds. Since it prevailed worldwide in 2003, poultry farmers have been under constant threat from loss of domestic birds due to the disease. The source of the disease is influenza virus H5N1 carried naturally by wild birds. Infection of domestic birds with H5N1 viruses leads to two types of diseases, low pathogenic form and highly pathogenic form. The infection of domestic birds with highly pathogenic form spreads rapidly over a poultry farm and causes domestic birds serious sicknesses that eventually lead to death. In practice, spot-check is conducted to detect infection of domestic birds with H5N1 viruses. If one bird in a farm is detected positive for infection, all the birds in the farm are disposed of.

Primary factors for outbreak of bird flu include avian influenza virus as source of disease, poultry as host, and environment as medium. In a production process on a poultry farm, the entire population of domestic birds is kept constant at the manageable capacity by supply of new healthy birds for vacancies. After intrusion of influenza virus, some infected birds die at an early stage of infection, and some others live longer. However, regardless of being alive or dead, infected birds are the sources of infection, unless they are completely removed from the entire population.

Mathematical models based on these factors were proposed to study bird flu infection processes within a poultry farm. A mathematical model originally proposed consists of ordinary differential equations whose unknowns are populations of susceptible birds and infected birds (Nova et al., 2010 (1)). The mathematical model was extended to cover time evolution of populations of susceptible birds and infected birds, and concentration of bird flu virus (Nova et al., 2010 (2), (3)).

Analysis based on the model has shown that the population of domestic birds can be made secure against infection with bird flu by proper vaccination and proper removal of infected birds. It is shown that the state free of infection is made stable in the sense that the state returns to the original state after intrusion of bird flu. It is also shown that the population cannot be made secure by vaccination alone without removal of infected bird and that it can be made secure by removal of infected birds alone without vaccination.

The study on bird flu infection processes within a poultry farm is continued with virus diffusion taken into consideration in modeling. In the following sections, the model is described and numerical results are introduced.

2 MODELING INFECTION PROCESSES WITHIN A POULTRY FARM

The intrusion of bird flu into a poultry farm divides the population of domestic birds into two classes, the class of healthy birds susceptible to infection and the class of infected birds.

The SI model (1), (2) is introduced in studies on infectious diseases to analyze the population of susceptible individuals x and the population of infected individuals y (Keeling et al., 2007).

$$\frac{dx}{dt} = c - bx - \omega xy, \tag{1}$$

$$\frac{dy}{dt} = \omega xy - (b+m)y. \tag{2}$$

Here c is the rate at which new birds are born, b is the death rate for susceptible birds and infected birds, and m is the additional death rate for infected birds. The term ωxy is the number of susceptible birds infected per unit time, and it is proportional to the number of susceptible birds x and the number of infected birds y. The system of equations (1), (2) is called the SI model because S and I stand for susceptible birds and infected birds, respectively, the population of which the unknowns of the system denote.

The SI model (1), (2) is not appropriate for closed systems such as poultry farms. In the production process of a poultry farm, the entire population of domestic birds is balanced with the capacity of the farm by supply of new healthy birds when

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vacancies are created, and the first two terms on the right hand side of the equation (1) are replaced with $a\{c-(x+y)\}$. Here c is the capacity of the farm, and a is the rate of supply of susceptible birds. The infected domestic birds eventually die from the disease. Some infected birds die at an early stage and others stay alive longer. Regardless of being alive or dead, infected birds remain as sources of infection unless they are removed from the population. The removal of infected birds is proportional to the population of infected birds, and so the second term in the right hand side of the equation (2) is replaced with $-my$, where m is the removal rate. The foregoing discussion leads to the system of differential equations (3) (Nova et al., 2010 (1)).

$$\begin{aligned} \frac{dx}{dt} &= a\{c-(x+y)\} - \omega xy\,, \\ \frac{dy}{dt} &= \omega xy - my\,. \end{aligned} \tag{3}$$

Here c, ω, and m are positive constants.

Stationary points of the system (3) are constant solutions obtained by setting the right hand sides equal to zero. For fixed but arbitrary positive constants a, c, ω, and m, there are two stationary points of the system (3). One stationary point of the system (3) is

$$(x,\,y) = (c,\,0)\ , \tag{4}$$

which corresponds to the state free of infection, in which none of the birds is infected. The other stationary point of the system (3) is

$$(x,\,y) = \left(\frac{m}{\omega},\,\frac{a\,(c\omega-m)}{\omega\,(a+m)}\right)\,. \tag{5}$$

The y component of the stationary point (5) is positive if and only if

$$c\omega - m > 0. \tag{6}$$

The stationary point (5) is practically significant under the condition (6), while it is practically insignificant for $c\omega - m < 0$. The stationary points (5) and (6) coincide for $c\omega - m = 0$.

When the stationary point (4) is asymptotically stable, the state always returns to the original state after a perturbation due to intrusion of bird flu. The stationary point (4) is unstable under the condition (6), and that it is asymptotically stable for $c\omega - m < 0$. The stationary point (5) is asymptotically stable under the condition (6), and that it is unstable for $c\omega - m < 0$ (Nova et al., 2010 (1)). The stability of a stationary point $(x,\,y) = (\xi,\,\eta)$ of the system (3) depends on the eigenvalues of the Jacobian matrix

$$A = \begin{bmatrix} -(a+\omega\eta) & -(a+\omega\xi) \\ \omega\eta & \omega\xi - m \end{bmatrix}.$$

The stationary point $(\xi,\,\eta)$ is asymptotically stable when all the eigenvalues of A have negative real parts, and unstable when at least one of the eigenvalues has a positive real part (Coddington et al., 1955). The eigenvalues λ_- and λ_+ of A are given by

$$\lambda_\pm = \frac{\operatorname{tr} A}{2} \pm \frac{\sqrt{(\operatorname{tr} A)^2 - 4\det A}}{2}. \tag{7}$$

Here

$$\operatorname{tr} A = -(a+\omega\eta) + \omega\xi - m, \tag{8}$$

and

$$\det A = -(a+\omega\eta)(\omega\xi - m) + \omega\eta\,(a+\omega\xi)\,. \tag{9}$$

The stationary point is asymptotically stable for tr A < 0 and $\det A > 0$.

For the stationary point (4), expressions (7) – (9) lead to

$$\lambda_- = -a, \qquad \lambda_+ = \omega c - m.$$

Under the condition (6), the stationary point (4) is unstable, and it is asymptotically stable for $c\omega - m < 0$. For the stationary point (5), expressions (8) and (9) become

$$\operatorname{tr} A = -\frac{a\,(a+c\omega)}{a+m}, \qquad \det A = a\,(c\omega - m)\,.$$

Under the condition (6), the stationary point (5) is asymptotically stable, and it is unstable under the condition $c\omega - m < 0$.

In order to propose effective measures against outbreaks of bird flu, it is important to take temporal and spatial distribution of virus concentration into consideration. The time rate in increase of virus concentration is proportional to itself. The decrease in number of susceptible birds due to infection is proportional to the population of susceptible birds, and it is also proportional to the virus concentration. The decrease in number of susceptible birds due to infection is the increase in number of infected birds. The rate of increase in virus concentration depends on the number of infected birds as hosts. It is positive when the virus concentration stays below the capacity of the hosts, and it becomes negative when the virus population exceeds the capacity of the hosts.

Let x, y, and z be the population of susceptible birds, the population of infected birds, and virus concentration, respectively. In the production process of a poultry farm, the total population $x + y$ is maintained at the capacity of the farm c. When vacancies are created $(c - (x + y) > 0)$, healthy susceptible birds are supplied. The increasing rate due to supply of healthy susceptible birds is $a\{c - (x + y)\}$, where a is a positive constant. When susceptible birds are infected with bird flu, they become infected birds. The number of susceptible birds infected per unit time is proportional to the virus concentration in the medium, and it is also proportional to the number of susceptible birds. The decreasing rate of susceptible birds due to infection is σxz. Here σ is a positive constant. The time rate of change in number of susceptible birds is the difference between the increasing rate and the decreasing rate, and

$$\frac{dx}{dt} = a\{c - (x + y)\} + \sigma xz$$

holds. The decreasing rate of the susceptible birds due to infection is the increasing rate of infected birds, the number of infected birds removed from the entire population is proportional to the number of infected birds, and

$$\frac{dy}{dt} = \sigma xz - my$$

holds. Infected birds are hosts of influenza virus. The increasing rate of the virus concentration is proportional to the number of infected birds, the decreasing rate of virus is proportional to the virus concentration, and

$$\frac{dz}{dt} = py - qz$$

holds. The forgoing discussion leads to the system of equations

$$\begin{aligned} \frac{dx}{dt} &= a\{c - (x + y)\} - \omega rxz, \\ \frac{dy}{dt} &= \omega rxz - my, \\ \frac{dz}{dt} &= p(y - rz)\,, \end{aligned} \tag{10}$$

which governs the time evolution of the population of susceptible birds, the population of infected birds, and the virus concentration (Nova et al., 2010 (2), (3)). Here $r = q/p$ and $\omega = \sigma/r$.

Stationary points of the system (10) are constant solutions obtained by setting the right hand sides equal to 0. For fixed but arbitrary positive constants a, c, ω, r, m, p, and q, there are two stationary points of system (10). One stationary point is

$$(x,\, y,\, z) = (c,\, 0,\, 0)\,, \tag{11}$$

which corresponds to the state free of infection. The other stationary point is

$$(x,\, y,\, z) = \left(\frac{m}{\omega},\, \frac{a(c\omega - m)}{\omega(a + m)},\, \frac{a(c\omega - m)}{r\omega(a + m)}\right). \tag{12}$$

The y component and z component of stationary point (12) are positive, and the stationary (12) point is practically significant if and only if the condition (5) holds. The stationary points (11) and (12) coincide for $m = c\omega$. As m increases from 0 to $c\omega$, stationary point (12) moves on a curve connecting the point $(0,\, c,\, c/r)$ and the stationary point (11) (Figure 1).

Suppose that (ξ, η, ζ) is a stationary point of the system (10). The stability of the stationary point (ξ, η, ζ) is determined by the eigenvalues λ_1, λ_2, and λ_3 of matrix

$$B = \begin{bmatrix} -(a + r\omega\zeta) & -a & -r\omega\xi \\ r\omega\zeta & -m & r\omega\xi \\ 0 & p & -pr \end{bmatrix}$$

The stationary point (ξ, η, ζ) is asymptotically stable when the real parts of λ_1, λ_2, and λ_3 are all negative, and it is unstable when at least one of the eigenvalues has a positive real part (Coddington et al., 1955). For the stationary point (11),

$$\lambda_1 = -a, \qquad \lambda_2 = -\frac{m + pr}{2} - \frac{\sqrt{(m - pr)^2 + 4pr\omega c}}{2}, \qquad \lambda_3 = -\frac{m + pr}{2} + \frac{\sqrt{(m - pr)^2 + 4pr\omega c}}{2}$$

These eigenvalues always have real values. For $0 \le m < \omega c$, λ_1 and λ_2 are negative, and λ_3 is positive. They are all negative for $m > \omega c$. The stationary point (11) is unstable for $0 \le m < \omega c$, and asymptotically stable for $m > \omega c$.

The eigenvalues of B associated with the stationary point (12) were evaluated for the special case $a = pr$ (Nova et al, 2010 (2)). In this case,

$$\lambda_1 = -a\,, \qquad \lambda_2 = -(a + m)\,, \qquad \lambda_3 = -\frac{a}{a + m}(c\omega - m)\,.$$

For $0 \le m < \omega c$, λ_1, λ_2, and λ_3 are all negative, and λ_1 and λ_2 are negative, and λ_3 is positive for $m > \omega c$. The stationary point (12) is asymptotically stable for $0 \le m < \omega c$, and it is unstable for $m > \omega c$.

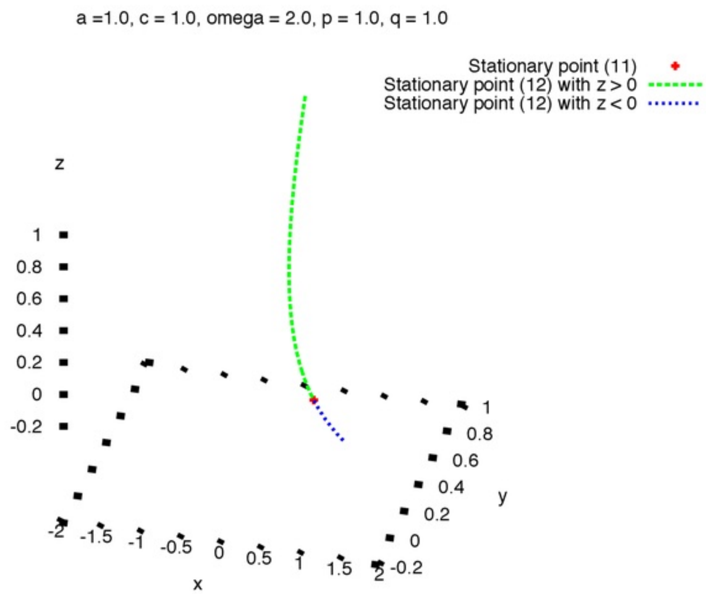

Figure 1: Stationary points of the system (10).

The system (10) was solved numerically for 100 initial values

$$x = i \times 0.5, \qquad y = j \times 0.5, \qquad z = k \times 0.5$$

for $i = 0, 1, 2, 3, 4$, $j = 0, 1, 2, 3, 4$, $k = 1, 2, 3, 4$ (Nova et al, 2010 (2), (3)) using the fourth-order Adams Bashforth-Moulton Predictor-Corrector in PECE mode in conjunction with the Runge-Kutta Method to generate values of approximate solution at the first three steps (Lambert, 1973). Here numerical results were obtained for $m = l \times 0.25$ ($l = 0, 1, \ldots, 20$) with time step length 0.001 (Nova et al., 2010 (3)). Here the system was solved numerically for 100,000 steps. Figure 2 shows the stationary points (11) and (12), and the numerical solutions for $a = 1$, $c = 1$, $\omega = 2$, $p = 1$, and $q = 1$, and for two different values of m, $m = 1$, and $m = 3$.

The influenza virus is transmitted through a medium, but the systems (3) and (10) reflect no spatial effects. Suppose that influenza viruses are transmitted through the medium by diffusion. Let ξ be the one dimensional coordinate variable (Figure 3). The system (10) becomes

$$\begin{aligned} \frac{\partial x}{\partial t} &= a\left\{c - (x + y)\right\} - \omega r x z, \\ \frac{\partial y}{\partial t} &= \omega r x z - m y, \\ \frac{\partial z}{\partial t} &= p\,(y - r z) + \lambda \frac{\partial^2 z}{\partial \xi^2}, \qquad 0 < \xi < l, \quad t > 0, \end{aligned} \tag{13}$$

The solution of the system is subject to the boundary condition

$$\frac{\partial z}{\partial \xi}(0, t) = \frac{\partial z}{\partial \xi}(l, t) = 0, \qquad t \geq 0, \tag{14}$$

and the initial condition

$$x(\xi, 0) = x_0(\xi), \qquad y(\xi, 0) = y_0(\xi), \qquad z(\xi, 0) = z_0(\xi), \qquad 0 < \xi < l. \tag{15}$$

Note that the stationary points of the system (10) are constant solutions of the system (13) that satisfy the boundary condition (14). The stabilities of the stationary solutions (11) and (12) as solutions of the boundary value problem (13), (14) are investigated in (3).

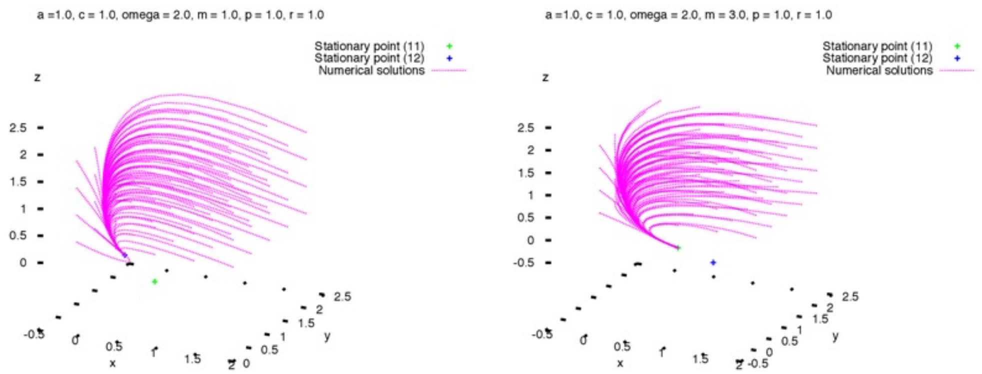

Figure 2: Numerical solutions. Numerical solutions of the system (10) are generated for $a = 1$, $c = 1$, $\omega = 2$, $p = 1$, $r = 1$, and for $m = 1$ and $m = 3$. All the numerical solutions terminated in the sphere of radius 10^{-3} centered at the stationary point (12) for $m = 1$ (left). All the numerical solutions terminated in the sphere of radius 10^{-3} centered at the stationary point (11) for $m = 3$ (right).

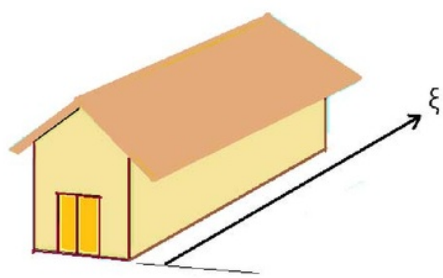

Figure 3: One dimensional spatial variable ξ.

3 NUMERICAL SIMULATION BASED ON HOST VIRUS MODEL

In order to solve the initial boundary problem, the interval $[0, 1]$ was divided into n intervals $[\xi_{i-1}, \xi_i]$ $(i = 1, 2, \ldots n)$ where $\xi_i = i\Delta\xi$ and $\Delta\xi = 1/n$, and the rectangular grid

$$(\xi_i, t_j) \qquad (i = 0, 1, \ldots n, \quad j = 0, 1, \ldots)$$

was set to implement a finite difference method. Denote by $x_i(t)$, $y_i(t)$, and $z_i(t)$ approximate values of $x(\xi_i, t)$, $y(\xi_i, t)$, and $z(\xi_i, t)$, respectively. When the central difference approximation is applied to the third equation of the system (13), the system (13) leads to

$$\begin{aligned} \frac{dx_i}{\partial dt} &= a\{c - (x_i + y_i)\} - \omega r x_i z_i\,, \\ \frac{dy_i}{dt} &= \omega r x_i z_i - m y_i\,, \qquad (i = 0, 1, \ldots, n) \end{aligned} \tag{16}$$

$$\frac{dz_i}{dt} = p(y_i - r z_i) + \lambda \frac{z_{i-1} - 2z_i + z_{i+1}}{\xi^2}. \qquad (i = 1, 2, \ldots, n-1) \tag{17}$$

The equation (17) holds for $i = 1, 2, \ldots, n-1$. For $i = 0$ and $i = n$, the boundary condition (14) implies

$$\frac{z_1 - z_{-1}}{\Delta\xi} = 0, \qquad \frac{z_{n+1} - z_{n-1}}{\Delta\xi} = 0,$$

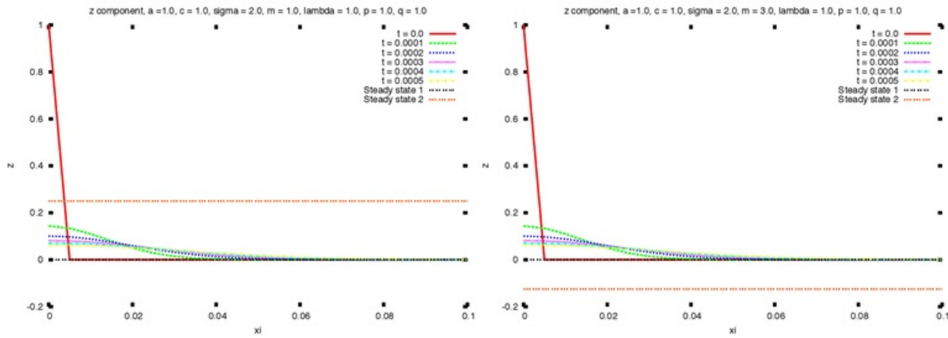

Figure 4: Numerical solution of the initial-boundary value problem (13) – (16). The figure shows the $z$component of the numerical solutions for $0 \leq t \leq 0.005$. The z component of the solution diffuses to become spatially uniform. There is no significant difference between numerical solutions for $m = 1$ and $m = 3$.

which leads to $z_{-1} = z_1$ and $z_{n+1} = z_{n-1}$. For $i = 0$ and $i = n$, the equation (17) becomes

$$\frac{dz_0}{dt} = p\left(y_0 - rz_0\right) + \lambda \frac{2z_1 - 2z_0}{\Delta\xi^2}, \qquad \frac{dz_n}{dt} = p\left(y_n - rz_n\right) + \lambda \frac{2z_{n-1} - 2z_n}{\Delta\xi^2}. \tag{18}$$

Suppose that intrusion of influenza virus takes place at $\xi = 0$. The initial boundary value problem (13), (14), (15) is solved numerically for $n = 200$, and

$$x_0\left(\xi\right) = c, \qquad y_0\left(\xi\right) = 0, \qquad z_0\left(\xi\right) = \begin{cases} 1 - 200\xi, & 0 \leq \xi < 0.005, \\ 0, & 0.005 \leq \xi < 1 \end{cases} \tag{19}$$

The initial value problem (16), (17), and (19) was solved numerically for $a = 1.0$, $c = 1.0$, $\sigma = 2.0$, $\lambda = 1.0$, $p = 1.0$, and $q = 1.0$, and two values of m, $m = 1$ and $m = 3$ using the Runge-Kutta Method (Lambert, 1973). Figure 4 shows the z component of the numerical solution for $m = 1$ and $m = 3$ over the period $0 \leq t \leq 0.005$. In both cases the solutions become spatially uniform at an early stage.

Figures (5) shows numerical solutions of the initial boundary value problem for $a = 1.0$, $\sigma = 2.0$, $m = 1.0$, $\lambda = 1.0$, $p = 1.0$, $q = 1.0$. over the period $0 \leq t \leq 15$. The figure shows that the solution of the initial boundary value problem approach uniformly to the stationary point (12). Figures (6) shows numerical solutions of the initial boundary value problem for $a = 1.0$, $\sigma = 2.0$, $m = 3.0$, $\lambda = 1.0$, $p = 1.0$, $q = 1.0$. over the period $0 \leq t \leq 15$. The figure shows that the solution of the initial boundary value problem approach uniformly to the stationary point (11).

4 DISCUSIION

Analysis based on a mathematical model shows that solutions of the initial boundary value problem approach to the stationary point (12) for small m, and that they approach the stationary point (11)for sufficiently large m. Figure 5) shows that the solution approach to the stationary point (12) uniformly after intrusion of bird flu for a small m, and the state does not return to the state free of infection. Figure 6) shows that the solution approach to the stationary point (11) uniformly after intrusion of bird flu for a sufficiently large m, and the state does return to the state free of infection. The results show that the removal of infected birds is essential for maintenance of the state free of infection. The state free of infection can be made secure against infection of bird flu by proper vaccination and proper removal of infected birds.

5 ACKNOWLEDGEMENTS

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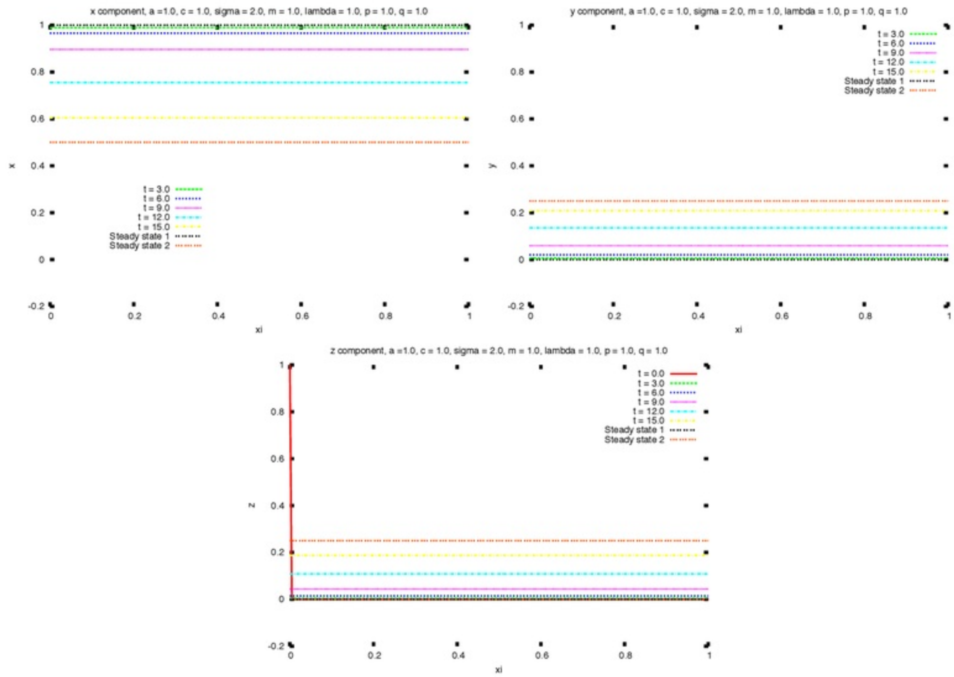

Figure 5: Numerical solutions of the initial boundary value problem (13) – (16). The figure shows the x component, y component, and $z$component of the numerical solutions for $a = 1.0$, $\sigma = 2.0$, $m = 1.0$, $\lambda = 1.0$, $p = 1.0$, $q = 1.0$. All the components converge uniformly to the corresponding components of the stationary point (12).

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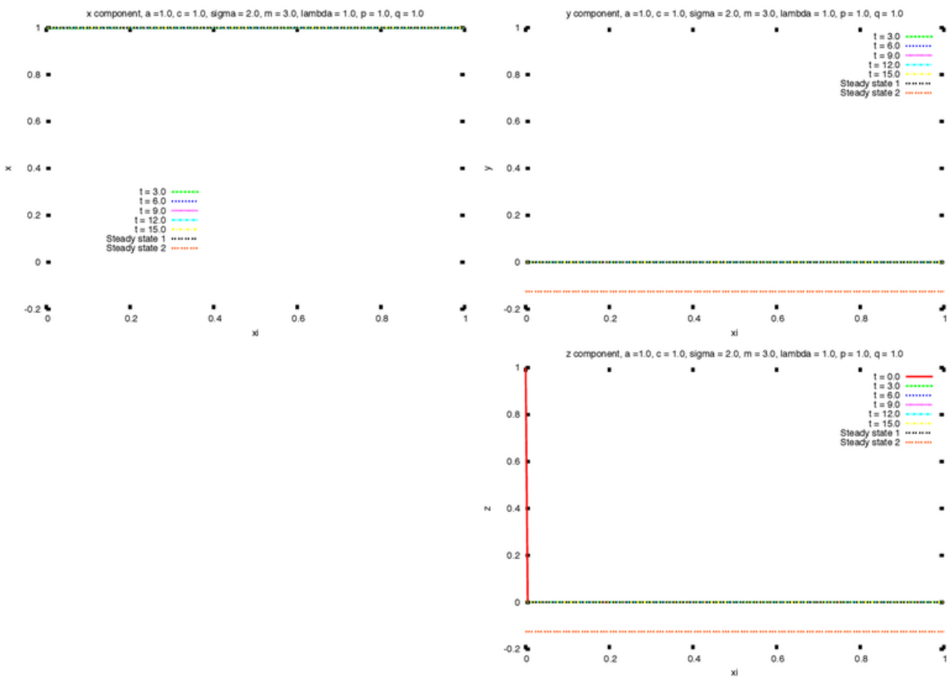

Figure 6: Numerical solutions of the initial boundary value problem (13) – (16). The figure shows the x component, y component, and $z$component of the numerical solutions for $a = 1.0$, $\sigma = 2.0$, $m = 3.0$, $\lambda = 1.0$, $p = 1.0$, $q = 1.0$. All the components converge uniformly to the corresponding components of the stationary point (11).

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