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Impulse elimination for positive singular systems using derivative output feedback

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Abstract. Nowadays the singular system constitute an interesting study considering it can become a mathematical model of variety of real problems. It is well known that the solutions of singular system can contain impulse. In some applications, this impulse constitutes some unwanted behaviour because it may cause degradation in the performance or even destroy the system. The positive singular system constitute a singular system in which its solution belong to some nonnegative octant. In this paper we propose a new technique to eliminate the impulses of the positive singular system using a derivative output feedback. Some new theorem that ensures the freedom of impulse of the positive singular system is established.

1. Introduction

Given the following singular system:

$$\begin{array}{r}E\dot{\mathbf{x}}(t)=A\mathbf{x}(t)+B\mathbf{u}(t)\,,\quad \mathbf{x}(0)=\mathbf{x}_0,\\ \mathbf{y}(t)=C\mathbf{x}(t)\end{array} \tag{1}$$

where $E, A\in \mathbb{R}^{n\times n}$, $B\in \mathbb{R}^{n\times m}, C\in \mathbb{R}^{p\times n}, \mathbf{x}\in \mathbb{R}^n$ denote state, $\mathbf{y}\in\mathbb{R}^p$ denote output vector, $\mathbf{u}\in\mathbb{R}^m$ denote input (control) and $\text{rank}(E)=r<n$. The system (1) is called the singular system. If $\text{rank}(E)=n$ then the first equation in (1) can be written as the following standard system:

$$\dot{\mathbf{x}}(t)=E^{-1}A\mathbf{x}(t)+E^{-1}B\mathbf{u}(t)\,,\quad \mathbf{x}(0)=\mathbf{x}_0. \tag{2}$$

The solution of (2) exists, unique and smooth for each initial state $\mathbf{x}_0$. If $\text{rank}(E)=r<n$, the system (1) has a solution if $\det(\alpha E-A)\neq 0$ for some $\alpha\in\mathbb{C}$ and for any admissible initial state. System (1) that satisfy the condition $\det(\alpha E-A)\neq 0$ for some $\alpha\in\mathbb{C}$ is called regular. Assume that $\det(\alpha E-A)\neq 0$ for some $\alpha\in\mathbb{C}$.

For a regular singular system (1), it is well known that there exist some nonsingular matrices Q_1 and P_1 such that

$$Q_1EP_1=\text{diag}\left[I_r,N\right],\ Q_1AP_1=\text{diag}(A_1,I_{n-r}),\ Q_1B=\begin{bmatrix}B_1\\B_2\end{bmatrix}\text{ and } CP_1=\begin{bmatrix}C_1 & C_2\end{bmatrix}, \tag{3}$$

where $B_1\in\mathbb{R}^{r\times m}$, $B_2\in\mathbb{R}^{(n-r)\times m}, C_1\in\mathbb{R}^{p\times r}, C_2\in\mathbb{R}^{p\times(n-r)}$ and N is nilpotent [2]. Thus, the system (1) is restricted equivalent to the following standard decomposition form:

$$\begin{aligned} \dot{\mathbf{x}}_1(t) &= A_1\mathbf{x}_1(t) + B_1\mathbf{u}(t) \\ N\dot{\mathbf{x}}_2(t) &= \mathbf{x}_2(t) + B_2\mathbf{u}(t) \\ \mathbf{y}(t) &= C_1\mathbf{x}_1(t) + C_2\mathbf{x}_2(t). \end{aligned} \tag{4}$$

If $N \neq O$, then the solution of (1) contains an impulsive term of the form

$$\mathbf{x}_2(t) = -\sum_{k=1}^{\nu-1} \delta^{(k-1)} N^k \mathbf{x}_2(0), \tag{5}$$

where δ is the delta Dirac function. Under this assumption, the solution of system (1) can exhibits impulse behaviour (not smooth) and usually this constitutes an unwanted behaviour because it may cause degradation in performance or even destroy the system. Therefore, it is important to eliminate the impulse for the singular system. Recently, the impulse elimination of the singular system was investigated by Li, et al. [4] and Zhang, et al. [7].

In the application fields in which the system (1) appears as a model of some real problems, the non-negativeness of the state variable is a must. A system, i.e. (1) or (2) , is called positive if for all $t \in \mathbb{R}_+$ we have $\mathbf{x}(t) \in \mathbb{R}_+^n$ and $\mathbf{y}(t) \in \mathbb{R}_+^p$ for any control $\mathbf{u}(t) \in \mathbb{R}_+^m$ and for any admissible initial state $\mathbf{x}_0 \in \mathbb{R}_+^n$ [3]. It well known that if $\text{rank}(E) = n$, system (2) is positive iff $E^{-1}A$ is a Metzler Matrix and $E^{-1}B \in \mathbb{R}_+^{n\times m}$. Another criterion for positiveness of the system (1) is given by Muhafzan [5] and Virnik [6]. In this paper we investigate the problem of impulse elimination for singular system (1) using derivative output feedback such that the resultant closed loop system is positive.

2. Problem Formulation

In Duan and Wu [2], it is stated that the impulse of the system (1) can be eliminated using a state feedback of the form $\mathbf{u}(t) = -K\mathbf{x}(t)$. Without positiveness constrain the system (1)is free impulse iff $\text{rank}\begin{bmatrix} E & B \end{bmatrix} = n$. We are interested in the problem of impulse elimination of singular system (1) using the following derivative output feedback

$$\mathbf{u}(t) = -K\dot{\mathbf{y}}(t) + \mathbf{v}(t), \tag{6}$$

where $K \in \mathbb{R}^{m\times p}$ is the gain matrix to be determined and $\mathbf{v}(t) \in \mathbb{R}_+^m$ is the new control with appropriate dimension. Applying the feedback (6) to system (1), the resultant closed loop system is obtained as

$$(E + BKC)\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{v}(t). \tag{7}$$

Consequently, our task now is to establish the criteria to ensure the resultant closed loop system (7) to be free impulse and positive.

To fulfill our task, we need the following definition and theorem that be referred to Canto, et al. [1]. Matrix $A \in \mathbb{R}^{n\times n}$ is said to be nonnegative if everyentry of A is nonnegative. Matrix $A \in \mathbb{R}^{n\times n}$ is said to be monotone if it is nonsingular and $A^{-1} \in \mathbb{R}_+^{n\times n}$. Matrix $M \in \mathbb{R}_+^{n\times n}$ is a monomial matrix if in every row and column of M there is exactly one nonzero entry. It has been proved that $A^{-1} \in \mathbb{R}_+^{n\times n}$ if and only if A is monomial.

Definition 1 [1]

(i) Two matrices $M \in \mathbb{R}_+^{n\times n}$ and $N \in \mathbb{R}_+^{n\times n}$ are called positively equivalent if there exist two monomial matrices P and Q such that $N = QMP$.

(ii) Given a matrix $M \in \mathbb{R}_+^{n\times n}$ *with* $\text{rank}(M) = r$. *The matrix* M *is* $r-$*monomial if it is positively equivalent to the matrix*

$$\begin{bmatrix} M_r & O \\ O & O \end{bmatrix}$$

where M_r *is monomial and* O *is the zero matrix of suitable size.*

The $r-$monomial matrices have the following properties.

Theorem 1 [1]

(i) The matrix M *is* $r-$*monomial iff* M *has* $n-r$ *rows and columns with all entries equal to zero and* r *rows and columns with only one entry different to zero.*

(ii) If the matrix M *is* $r-$*monomial, then it is positively equivalent to the matrix*

$$\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}.$$

3. Main Result

Assume that the matrix E is positively equivalent to $r-$monomial matrix, that is

$$QEP = \begin{bmatrix} M_r & O \\ O & O \end{bmatrix}. \quad (8)$$

Moreover, without loss of generality we assume that

$$QB = \begin{bmatrix} O \\ B_2 \end{bmatrix} \text{ and } CP = \begin{bmatrix} C_1 & C_2 \end{bmatrix}. \quad (9)$$

The condition $\text{rank}\begin{bmatrix} E & B \end{bmatrix} = n$ is equivalent to B_2 has full row rank.

We have the following result.

Theorem 2 *Consider the system* (1) *with* $A \in \mathbb{R}_+^{n\times n}$, E, B *and* C *are given in* (8) *where* B_2 *and* C_2 *have full rank. Then the system* (1) *is free impulsive and positive if:*

(i) $\left(B_2B_2^TC_2^TC_2\right)^{-1} \in \mathbb{R}_+^{(n-r)\times(n-r)}$

(ii) $B_2B_2^TC_2^TC_1 \in \mathbb{R}_-^{(n-r)\times r}$

(iii) $B_2 \in \mathbb{R}_+^{(n-r)\times m}$.

Proof. Using (8), (9) and results in Virnik [1], we can choose $K = B_2^TC_2^T$ such that

$$\begin{aligned} E + BKC &= Q^{-1}\begin{bmatrix} M_r & O \\ O & O \end{bmatrix}P^{-1} + Q^{-1}QBKCPP^{-1} \\ &= Q^{-1}\begin{bmatrix} M_r & O \\ O & O \end{bmatrix}P^{-1} + Q^{-1}\begin{bmatrix} O \\ B_2 \end{bmatrix}B_2^TC_2^T\begin{bmatrix} C_1 & C_2 \end{bmatrix}P^{-1} \\ &= Q^{-1}\left(\begin{bmatrix} M_r & O \\ O & O \end{bmatrix} + \begin{bmatrix} O & O \\ B_2B_2^TC_2^TC_1 & B_2B_2^TC_2^TC_2 \end{bmatrix}\right)P^{-1} \\ &= Q^{-1}\begin{bmatrix} M_r & O \\ B_2B_2^TC_2^TC_1 & B_2B_2^TC_2^TC_2 \end{bmatrix}P^{-1}. \end{aligned}$$

Since B_2 and C_2 have full rank, the matrix $E+BKC$ is nonsingular, hence

$$\begin{aligned}(E+BKC)^{-1} &= P\begin{bmatrix} M_r^{-1} & O \\ -\left(B_2B_2^TC_2^TC_2\right)^{-1}B_2B_2^TC_2^TC_1M_r^{-1} & \left(B_2B_2^TC_2^TC_2\right)^{-1}\end{bmatrix}Q \\ &= P\begin{bmatrix} M_r^{-1} & O \\ O & \left(B_2B_2^TC_2^TC_2\right)^{-1}\end{bmatrix}\begin{bmatrix} I & O \\ -B_2B_2^TC_2^TC_1M_r^{-1} & I\end{bmatrix}Q.\end{aligned}$$

If the condition (1) and (2) hold, then $(E+BKC)^{-1}$ is nonnegative and thus $(E+BKC)^{-1}A$ is nonnegative. Moreover, conditions (3) implies $(E+BKC)^{-1}B$ is nonnegative. Hence, the control (6) allows us to get a positive standard system. ■

Note that the condition for the matrix $A\in\mathbb{R}_+^{n\times n}$ in Theorem 2 is very tight. Modification of Theorem 2 in which $A\in\mathbb{R}^{n\times n}$ is in preparation.

4. Conclusion

A new technique to eliminate the impulses of the positive singular system using a derivative output feedback has been established. A new theorem which ensures freedom of impulses of the positive singular system has been proved.

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